Introduction to Cosmology

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- Modern cosmology started with Einstein's publication of the General Relativity Theory (GR) ca. 1915

General Relativity

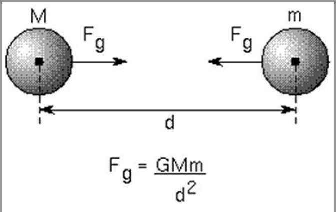

Figure: Newton's universal gravitation (1687). Gravity as force-at-a-distance. [Credit: physics.stackexchange.com]

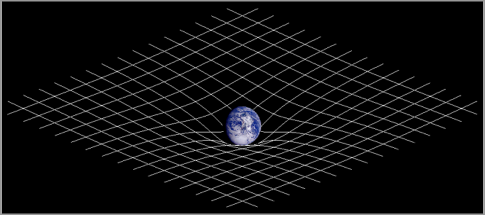

Figure: Einstein's GR (1915). Gravity as curvature of spacetime. "Matter tells spacetime how to curve, spacetime tells matter how to move."

The Expanding Universe

- Einstein's field equations of GR predict an expanding universe
- Einstein added the cosmological constant to make the universe static
- Observations confirmed an expanding universe (1920s)
- Einstein commited " the biggest blunder of my scientific career"

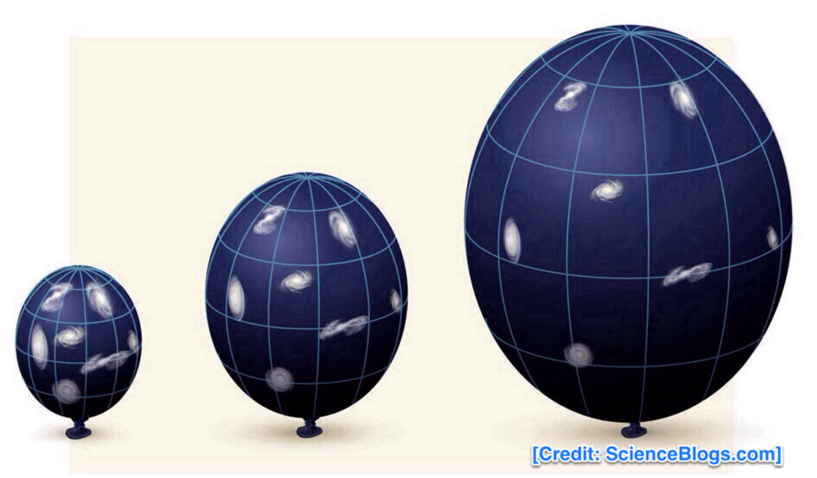

Figure: The expanding Universe

What Does Gravity Have to Do with It?

Interaction	Current Theory	Mediators	Relative Strength	Long-Distance Behavior	Range (m)
Strong	Quantum chromodynamics (QCD)	gluons	10^{38}	1	10^{-15}
Electromagnetic	Quantum electrodynamics (QED)	photons	10^{36}	$\frac{1}{r^2}$	∞
Weak	Electroweak Theory	W and Z bosons	10^{25}	$\frac{d}{dr}\left(\frac{\exp(-m_{W,Z}r)}{r}\right)$	10^{-18}
Gravitation	General Relativity (GR)	gravitons (hypothetical)	1	$\frac{1}{r^2}$	∞

Figure: The fundamental forces of nature.

What does gravity have to do with it?...

- Based on Einstein's field equations (EFEs) derived from the Einstein-Hilbert action[1]

$$\mathcal{A} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g}\,[R + 2(\mathcal{L}_m - \Lambda)] \tag{1}$$

$$G_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab} \tag{2}$$

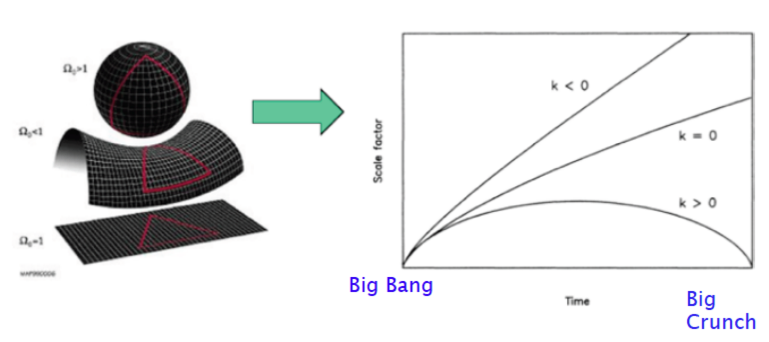

Figure: Geometry and evolution ...

[1] Customary to set $c = 8\pi G = 1$.

The Cosmological Crisis

- Astronomical observations suggest the expansion of the Universe is speeding up (1998)
- No satisfactory explanation yet

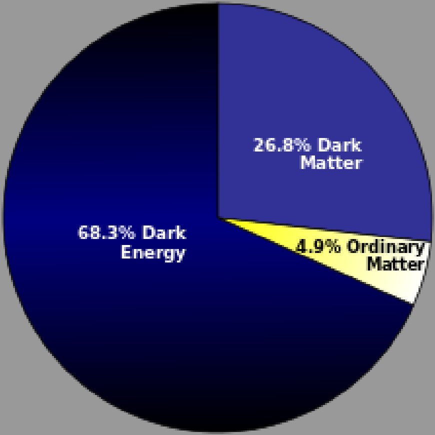

Figure: The cosmic pie crisis

- Several alternative theories/cosmological models proposed...

The Concordance Cosmological Model

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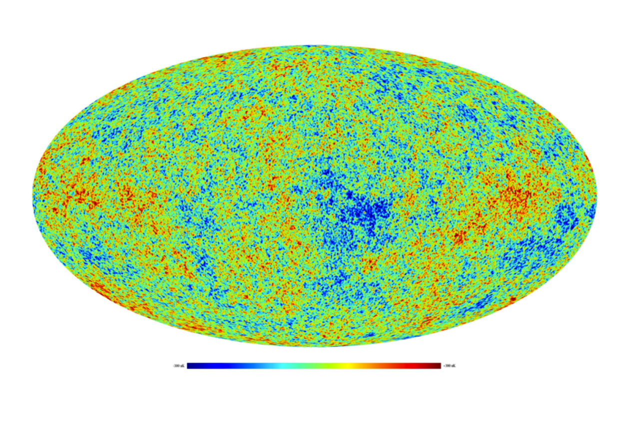

Figure: The Cosmic Microwave Background (CMB). Today, $T \sim 2.726K$, $\frac{\delta T}{T} \sim 10^{-5}$. [Credit: www.esa.int]

- Based on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric
- Describes a globally homogenous, isotropically expanding (or contracting) spacetime geometry

$$ds^2 = -dt^2 + a^2(t)\left[d^2r + f^2(r)(d^2\theta + \sin^2\theta d^2\phi)\right] \tag{3}$$

- $a(t) \equiv$ the cosmological scale factor, a time-dependent parameter that tells us the relative size of the Universe.

$$f(r) = \begin{cases} \sin(r) & \text{for } k = +1, \\ r & \text{for } k = 0, \\ \sinh(r) & \text{for } k = -1. \end{cases}$$

- The worldlines with tangent vector

$$u^a = \frac{dx^a}{dt} \tag{4}$$

  represent the normalized 4 - velocity of fundamental observers

- The $t = constant$ space sections are surfaces of homogeneity and have maximal symmetry: they are 3-spaces of constant curvature

$$K = \frac{k}{a^2(t)}\ , \tag{5}$$

  where k is the sign of K and hence takes the values $-1, 0$ or $+1$ depending on whether the Universe is open, flat or closed, respectively.

- The rate of cosmic expansion at any time t is characterized by the Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{6}$$

- The redshift z of an object emitting a wavelength λ_e (and a frequency ν_e) and observed with wavelength λ_o (and a corresponding frequency ν_o) is defined to be the fractional Doppler shift of its emitted light (photons) due to its radial motion:

$$z \equiv \frac{\lambda_o}{\lambda_e} - 1 = \frac{\nu_e}{\nu_o} - 1 = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \qquad (\text{where } \beta \equiv \frac{v}{c}) \tag{7}$$

$$z = \frac{a_0}{a(t)} - 1 \tag{8}$$

$*$ Exercise: Use Taylor expansion to show that, for $z \ll 1$,

Hubble's Law

$$v = H_0 D \tag{9}$$

with the 'distance' ('naive Hubble distance') D to a galaxy and v its velocity of recession. H_0 is Hubble's original proportionality constant, today known as the 'Hubble constant'.

The Hubble Constant

- One of the most important cosmological parameters
- SI unit (show!) is s^{-1}, but values are usually quoted in $km/s/Mpc$
- 1 $parsec \simeq 3.086 \times 10^{16}$ $meters \simeq 3.262$ $ly \rightarrow$ distance at which one astronomical unit subtends an angle of one arcsecond

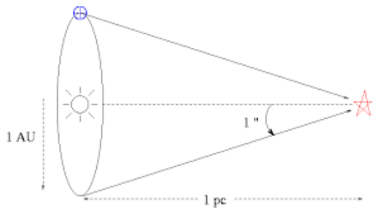

Figure: [Credit: spiff.rit.edu]

- $km/s/Mpc \rightarrow$ the speed in km/s of a galaxy 1Mpc away
- Hubble's initial value: $H_0 \sim 500$ $km/s/Mpc \sim 160 km/s$ per $million$ $light-years$
- Common notation: $H_0 = 100h$ $km/s/Mpc$, $h \rightarrow$ uncertainty parameter
- H_0 sets a time scale H_0^{-1}, and together with the matter and energy content of the Universe, sets the age of the Universe.

Date published	Hubble constant (km/s)/Mpc	Observer
2013-03-21	67.80 ± 0.77	Planck Mission
2012-12-20	69.32 ± 0.80	WMAP (9-years)
2010	$70.4^{+1.3}_{-1.4}$	WMAP (7-years), combined with other measurements.
2010	71.0 ± 2.5	WMAP only (7-years).
2009-02	70.1 ± 1.3	WMAP (5-years). combined with other measurements.
2009-02	$71.9^{+2.6}_{-2.7}$	WMAP only (5-years)
2006-08	$77.6^{+14.9}_{-12.5}$	Chandra X-ray Observatory
2007	$70.4^{+1.5}_{-1.6}$	WMAP (3-years)
2001-05	72 ± 8	Hubble Space Telescope
prior to 1996	50—90 (est.)	
1958	75 (est.)	Allan Sandage

Figure: [Credit: WIKI]

Dynamics of Cosmic Expansion

- The EFEs show the effect of matter on space-time curvature.
- Matter and energy are sourced by the Energy-Momentum Tensor (EMT) T_{ab} , which for perfect-fluid FLRW universes is given by

$$T_{ab} = \mu u_a u_b + p h_{ab} \ , \tag{10}$$

where

$$h_{ab} = g_{ab} + u_a u_b \tag{11}$$

is the projection tensor into the tangent 3-spaces orthogonal to u^a.

- The energy density and the pressure terms $\mu(t)$ and $p(t)$ are the time - like and space - like eigenvalues of T_{ab}, respectively.
- The evolution of the energy density gives the conservation equation

$$T^{ab}{}_{;b} = 0 \Leftrightarrow \dot{\mu} + (\mu + p)\Theta = 0 \ , \tag{12}$$

where $\Theta \equiv 3H$.

- Equation of state

$$w = \frac{p}{\mu} \tag{13}$$

- For example, Cold Dark Matter (**dust**) is pressureless and hence $w_d = 0$ whereas radiation has

$$p_r = \mu_r/3 \Leftrightarrow w_r = 1/3 \tag{14}$$

- Photon energy density

$$\mu_r \propto T^4 \tag{15}$$

$*$ Exercise: Using the conservation equation (12), show that

$$\mu_d \propto a^{-3}, \quad \mu_r \propto a^{-4}, \quad T \propto a^{-1}. \tag{16}$$

- We can also think of the cosmological constant Λ as a fluid such that

$$p_\Lambda = -\mu_\Lambda \tag{17}$$

and hence with a corresponding equation of state $w_\Lambda = -1$.
$*$ Exercise: How does the energy density of Λ grow/decay with the scale factor a?

The Friedmann Equation

- Shows how matter directly causes a curvature of 3-spaces:

$$\frac{\dot{a}^2}{a^2} \equiv H^2 = \frac{\mu}{3} + \frac{\Lambda}{3} - \frac{k}{a^2}. \tag{18}$$

- Controls the expansion of the Universe, and the conservation equation (12) controls the density of matter as the Universe expands
- Note here that μ is the total energy density of all kind of matter in the Universe. For example, if we include baryons (b), radiation (r), Cold Dark Matter (CDM) and neutrinos (ν), then

$$\mu = \mu_b + \mu_r + \mu_{CDM} + \mu_\nu \ , \tag{19}$$

  and this requires the specification of the equation of state w_i for each component i of matter
- Since CDM is by far the dominant component in a baryon-CDM mixture, it is customary to approximate the mixture as pressureless *dust*:

$$\mu_d \equiv \mu_b + \mu_{CDM} \tag{20}$$

- It is also customary to combine dust and radiation fluids as *standard matter* and write:

$$\mu_m \equiv \mu_d + \mu_r \tag{21}$$

∗ Exercise Show that the Friedmann equation can be written in a more compact form as

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1 \tag{22}$$

where Ω_i is a normalized dimensionless density parameter for the i-th cosmic fluid component.

The Raychaudhuri Equation

- Gives the basic evolution equation for the scale factor $a(t)$

$$3\frac{\ddot{a}}{a} = -\tfrac{1}{2}(\mu + 3p) + \Lambda \,, \tag{23}$$

  and hence the basic equation of gravitational interactions and the basis of singularity theorems in GR
- Shows that the active gravitational mass density of the matter and fields present is

$$\mu_{grav} \equiv \mu + 3p \,. \tag{24}$$

- For ordinary matter the *Strong Energy Condition* (SEC) imposes a positive gravitational mass density according to

$$\mu + 3p > 0 \Leftrightarrow w > -1/3 \,. \tag{25}$$

- Ordinary matter will tend to cause the Universe to decelerate ($\ddot{a} < 0$) whereas a positive cosmological constant, according to Eqn (23), causes an accelerated expansion ($\ddot{a} > 0$).
- Deceleration parameter

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} \tag{26}$$

Initial Conditions

- Ensure the existence of unique cosmological solutions
- At an arbitrary time t_0 (say, today) [2], initial data for such solutions consists of :

  The Hubble constant

$$H_0 = \left[\frac{\dot{a}}{a}\right]|_{t_0} = 100h \ \ \text{km/sec/Mpc} \tag{27}$$

  A dimensionless normalized density parameter

$$\Omega_{i0} = \frac{\mu_{i0}}{3H^2} \tag{28}$$

  for each type of matter present;

  For a non-vanishing cosmological constant , i.e., $\Lambda \neq 0$, either the fractional energy density

$$\Omega_{\Lambda 0} = \frac{\Lambda}{3H_0^2} \tag{29}$$

  The dimensionless *deceleration parameter*

$$q_0 = -\left[\frac{\ddot{a}}{a}\right]|_{t_0} H_0^{-2} \tag{30}$$

- Determine unique corresponding cosmic history

[2] Here, however, a subscript 0, unless otherwise indicated, will refer to the present epoch.

- If the pressure term p is negligible relative to the matter term μ in (23), then we get (show!)

$$q_0 = \frac{1}{2}\Omega_{m0} - \Omega_{\Lambda 0} \; . \tag{31}$$

- (31) $\implies$ a dominant cosmological constant ($\Omega_{\Lambda 0} > \Omega_{m0}$) causes an accelerated cosmic expansion ($q_0 < 0$).
- Matter can cause deceleration ($q > 0$) of the expansion if $\Omega_{m0} > 2\Omega_{\Lambda 0}$; in particular,

$$\Lambda = 0 \implies q_0 = \frac{1}{2}\Omega_{m0} \implies \text{deceleration} \tag{32}$$

- The spatial curvature

$$K_0 = \frac{k}{a_0^2} = H_0^2(\Omega_0 - 1) \tag{33}$$

can be obtained by evaluating the Friedmann equation (18) at the present time t_0. The Universe is said to be

$$\begin{cases} \text{open} & \text{if } K_0 < 0 \implies \Omega_0 < 1 \\ \text{flat} & \text{if } K_0 = 0 \implies \Omega_0 = 1 \\ \text{closed} & \text{if } K_0 > 0 \implies \Omega_0 > 1 \end{cases}$$

where $\Omega_0 \equiv \Omega_{d0} + \Omega_{\Lambda 0}$

Solutions

- The FLRW models are the most widely explored cosmological models
  - Extremely simple geometry
  - Ever-increasing accuracy of supporting observational data
- Defining $\rho_D \equiv \mu_{d0} a_0^3$ and $\rho_R \equiv \mu_{r0} a_0^4$ such that $\dot{\rho}_D = 0$ and $\dot{\rho}_R = 0$, one can rewrite Eqn (18) for dust and non-interacting radiation as

$$3\frac{\dot{a}^2}{a^2} = \frac{\rho_D}{a^3} + \frac{\rho_R}{a^4} + \Lambda - 3\frac{k}{a^2} \,. \tag{34}$$

For a vanishing cosmological constant $(\Lambda = 0)$

- The Universe starts off at a very dense initial state, where its energy density and curvature tend to infinity
- Future fate depends on the value of the spatial curvature, or equivalently the density parameter Ω_0.
- Expands forever if $k = 0 \Leftrightarrow \Omega_0 = 1$ or $k < 0 \Leftrightarrow \Omega_0 < 1$
- Collapses to a future singularity if $k > 0 \Leftrightarrow \Omega_0 > 1$
- $\Omega_0 = 1$ corresponds to the critical density μ_{crit} separating $\Lambda = 0$ FLRW models that recollapse in the future from those that expand forever, and Ω_0 is just the ratio of the matter density to this critical density:

$$\Omega_{crit} = 1 \Leftrightarrow \mu_{crit} = 3H_0^2 \Rightarrow \Omega_0 = \frac{\mu_0}{3H_0^2} = \frac{\mu_0}{\mu_{crit}} \tag{35}$$

When $\Lambda < 0$

- All solutions start at a singularity and recollapse.

When $\Lambda > 0$, there are some interesting possible scenarios:

- If $k = 0$ or $k = -1$, all solutions start at a singularity and expand forever.
- If $k = +1$, there can be
  - models with a singular start, either expanding forever or collapsing to a future singularity
  - a static solution (the Einstein static universe)
  - models asymptotic to Einstein static state in either the future or the past.
- Models with $k = +1$ can bounce (collapsing from infinity to a minimum radius and re-expanding).

Some very specific models with simple expanding solutions

- The Einstein-de Sitter model
  - This is the simplest $(p = 0, \Lambda = 0, k = 0)$ $(\Rightarrow \Omega_0 = 1)$ expanding non-empty solution

$$a(t) = Ct^{2/3} \quad (\text{show!}) \ , \tag{36}$$

  where C is an integration constant.
  - This solution starts from a singular state at time $t = 0$.
  - The age of the Universe in this model (the proper time since the start of the Universe) when the Hubble constant takes the value H_0 is (show!)

$$\tau_0 = \frac{2}{3H_0} \tag{37}$$

- The Milne model
  - Characterized by $(\mu = p = 0, \Lambda = 0, k = -1) \Rightarrow \Omega_0 = 0$ and represents a linearly expanding empty solution

$$a(t) = Ct \quad (\text{show!}) \; , \tag{38}$$

  in a flat spacetime as seen by a uniformly expanding set of observers, singular at $t = 0$.
  - The age of the Universe in this model is given by (show!)

$$\tau_0 = \frac{1}{H_0} \tag{39}$$

- The de Sitter universe
  - Characterized by $(\mu = p = 0, \Lambda \neq 0, k = 0) \Rightarrow \Omega_0 = 0$
  - The Universe is in a steady state of expansion without matter, the empty solution given by

$$a(t) = Ce^{Ht} \; , \quad (\text{show!}) \tag{40}$$

  where C and H are constants
  - The Universe expands at a constant rate in this model, and hence there is no start and its age is infinite (show!)

∗ Exercise: Show that a de Sitter universe with $k = +1$ has the bounce solution given by

$$a(t) = a_0 \cosh(Ht) \tag{41}$$

whereas that with $k = -1$ has the singular start solution

$$a(t) = a_0 \sinh(Ht) \; . \tag{42}$$

Cosmography

- Science of measuring the 'distance' between two observed cosmological objects or events
- Challenging: lack of accurate cosmological data + cosmic expansion
- Use direct observable such as the luminosity of a quasar, the redshift of a galaxy, or the angular size of the CMB power spectrum acoustic peaks to indirectly measure another quantity not directly observable, but mathematically calculable, such as the comoving coordinates of the quasar or the galaxy.
- Cosmological distance measures (and ages) largely exploit the fact that light travels on radial null geodesics with tangent vectors

$$k^a = \frac{dx^a}{d\lambda} \; , \tag{43}$$

satisfying the (null) conditions

$$k^a{}_{;b} k^b = 0, \;\; k^a k_a = 0. \tag{44}$$

- If a photon is emitted at a time t_e , the comoving radial distance $r(t_e, t_o)$ it travels before it is received by an observer (receiver) at a later time t_0 is given, according to the metric defined by Eqn (3),

$$ds^2 = 0, \tag{45}$$

with

$$d\theta = 0 = d\phi, \tag{46}$$

and hence

$$r(t_e, t_o) = \int_{t_e}^{t_o} \frac{dt}{a} = \int_{a_e}^{a_o} \frac{da}{a\dot{a}}. \tag{47}$$

- The Hubble time given by $\tau_H = \frac{1}{H_0}$ is the time taken for light to traverse a Hubble distance

$$D_H = \frac{c}{H_0} \ . \tag{48}$$

- Based on current H_0 estimates coming from:
  - recession velocities of objects around us

$$H_0 = 73 \pm 2.4 \mathrm{kms}^{-1} Mpc^{-1} \tag{49}$$

  - the 9-year WMAP analysis

$$H_0 = 68.65 \pm 0.93 \mathrm{kms}^{-1} Mpc^{-1} \tag{50}$$

  - Planck satellite's most recent results

$$H_0 = 67.3 \pm 1.2 \mathrm{kms}^{-1} Mpc^{-1} \tag{51}$$

we get

$$\tau_H \simeq 1.2 - 1.5 \times 10^{10} \mathrm{years} \tag{52}$$

$$D_H \simeq 1.2 - 1.5 \times 10^{26} \mathrm{m} \simeq 3700 - 4700 \mathrm{Mpc} \tag{53}$$

- Normalized to the geometric units:

$$\tau_H = D_H = c = 1 \tag{54}$$

Redshift

- The redshift z of an object emitting a wavelength λ_e (and a frequency ν_e) and observed with wavelength λ_o (and a corresponding frequency ν_o) is defined to be the fractional Doppler shift of its emitted light (photons) due to its radial motion:

$$z \equiv \frac{\lambda_o}{\lambda_e} - 1 = \frac{\nu_e}{\nu_o} - 1 \tag{55}$$

- The redshift of a moving object in an expanding universe can be given by

$$1 + z = (1 + z_c)(1 + z_v), \tag{56}$$

where z_v is the redshift due to the local peculiar motion of the object whereas z_c is the cosmological redshift due to the expansion of the Universe given in terms of the scale factor as

$$1 + z_c = \frac{a(t_0)}{a(t_e)}. \tag{57}$$

- For comoving objects, we have $z_v = 0$ from which follows $z_c = z$.

Distance Types

- The *proper distance* D_p

$$D_p = \int dt = \int_0^z \frac{dz'}{(1+z')H(z')} = \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')h(z')} \tag{58}$$

where $h(z) = H(z)/H0$ and in the FLRW model with Λ,

$$h(z) = \sqrt{\Omega_r(1+z)^4 + \Omega_d(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}\,. \tag{59}$$

- The *line-of-sight comoving distance* D_c between two nearby objects in the Universe is the constant distance between them at any epoch if the two objects are moving with the Hubble flow:

$$D_c = \int \frac{cdt}{a} = D_H \int_0^z \frac{dz'}{h(z')} \tag{60}$$

- The *transverse comoving distance* D_M measures the distance between two events at the same redshift but separated by a certain angle in the sky:

$$D_M = \begin{cases} D_H \frac{1}{\Omega_k}\sinh(\sqrt{\Omega_k}D_c/D_H) & \text{for } \Omega_k > 0 \\ D_c & \text{for } \Omega_k = 0 \\ D_H \frac{1}{|\Omega_k|}\sinh(\sqrt{|\Omega_k|}D_c/D_H) & \text{for } \Omega_k < 0 \end{cases}$$

- The *angular diameter distance* D_A defines the ratio of an object's transverse physical size to its radian angular size:

$$D_A = \frac{\ell}{\Delta\theta} \tag{61}$$

- Relationship between the angular diameter distance and the comoving distance:

$$D_M = (1+z)D_A. \tag{62}$$

- The *luminosity distance* D_L

$$D_L \equiv \sqrt{\frac{L}{4\pi f}}, \tag{63}$$

relates the *luminosity* L and the *flux* f of a distant object such as a supernova

- Related to other distances via the "distance duality" relation:

$$D_L = (1+z)D_M = (1+z)^2 D_A \tag{64}$$

- *Lookback time* t_L : the difference between the present age of the Universe t_0 and the age t_e of the Universe when the photons were emitted as measured by a hypothetical observer attached to the object

$$t_L = t_H \int_0^z \frac{dz'}{(1+z')h(z')} \tag{65}$$

- The *light travel distance LTD* is the distance the emitted photons travel during the lookback time:

$$LTD = ct_L. \tag{66}$$

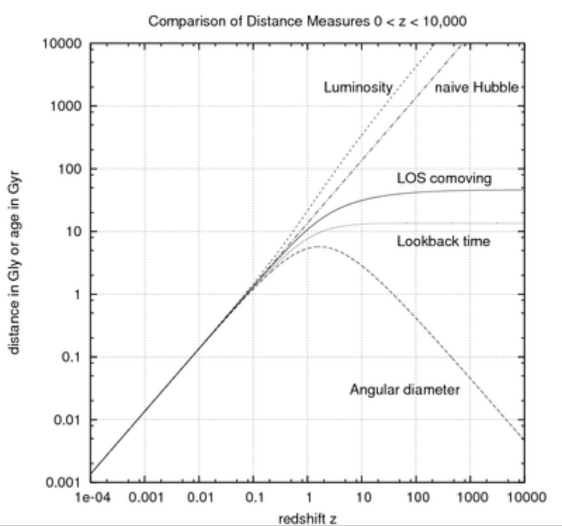

Figure: Distance measures of the nearby observable universe in the ΛCDM cosmology for $\Omega_{d\mathbf{0}} = 0.266, \Omega_{r\mathbf{0}} \simeq 7.7. \times 10^{-\mathbf{5}}, \Omega_{\Lambda\mathbf{0}} \simeq 0.732, \Omega_{k\mathbf{0}} \simeq 0.0019$. Geometrized units have been used with $H_{\mathbf{0}} = 72km/s/Mpc$. Note that the angular diameter distance and the light travel distance (or the lookback time) coincide in this plot. [Credit: WIKI]

Horizons in cosmology

- Finite speed of light → restricted causal relationships ∵ causal effects cannot propagate faster than light
  - Constrained features of cosmological structure formation and our observational knowledge of the cosmos
  - We can only influence or be influenced by regions inside our past null cone
  - There are regions of the Universe beyond which we have no access

Horizon

The boundary separating the accessible part of the Universe from the inaccessible

- The *particle horizon*: maximum distance particles could move to an observer during the Universe's period of existence; largest region of spacetime we could have probed so far

$$\chi_{ph} = \int_0^{t_0} \frac{dt'}{a(t')} \tag{67}$$

- The *event horizon*: largest comoving distance that light emitted now can ever reach an observer any time in the future; sets the maximum extent to the particle horizon

$$\chi_{eh} = \int_{t_0}^{t_{\inf}} \frac{dt'}{a(t')} \tag{68}$$

  - Blackholes → escape velocity of blackholes inside the horizon is superluminal → light emitted from beyond the horizon can never reach the observer and time itself stops at the boundary
- The *comoving Hubble radius* $\lambda_H = 1/aH$ determines the relevant physical scales for local causal influences in an expanding universe and increases during any standard evolutionary history of the Universe (such as the radiation-dominated and dust-dominated epochs)

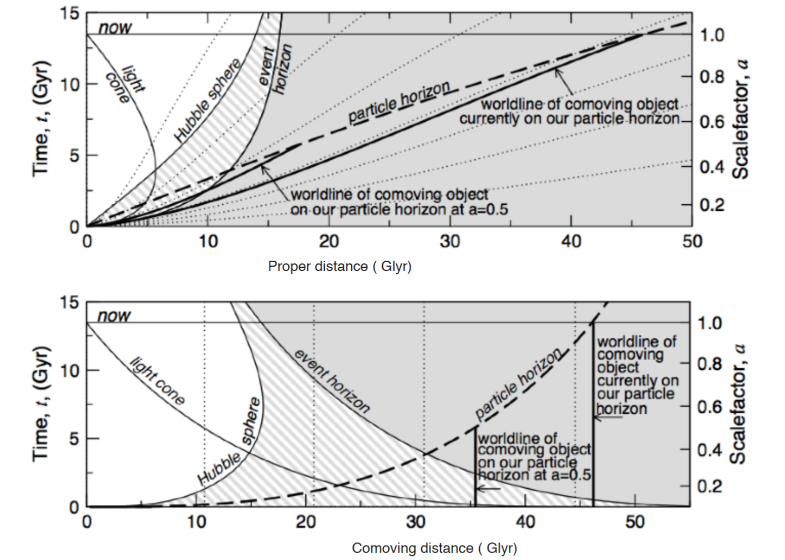

Figure: Horizons in ΛCDM cosmology [?, ?] for $\Omega_{m0} = 0.3, \Omega_{\Lambda 0} = 0.7, h = 0.7$. Top panel shows proper distance $D_p = a\chi_{ph}$ and bottom panel shows comoving distance $D_c = a_0\chi_{ph}$.

Shortcomings of the Standard Model

- The Hot Big Bang model is by far the most successful cosmological paradigm in explaining
  - the expansion of the Universe
  - the origin of the CMB
  - the synthesis of light elements
  - the formation of galaxies and large-scale structure
- It leaves many serious puzzles unanswered, some of them originating in the early Universe, some appearing in the late-time Universe
  - The Horizon Problem : why do causally disconnected regions in the Universe share similar physical properties such as temperature?
  - The Flatness Problem : fine-tuning of the initial conditions which would otherwise have greatly affected the geometry and expansion history of the Universe
  - The Structure Problem (Smoothness Problem/Homogneity Problem) : what brought about the matter clumping that finally led to cosmic structures like galaxies and clusters?
  - The Anti-Matter Problem (Baryon Asymetry Problem) : at high enough temperatures $kT \geq m_p c^2$, where m_p is the mass of a proton, there are roughly equal numbers of photons (γ), protons (p) and antiprotons ($\bar{p}$) in equilibrium, whereas the ratios today stand at $N_p/N_\gamma \sim 10^{-9}$ and $N_{\bar{p}}/N_p \sim 0$. Since baryon number is a conserved quantity, it would then necessarily imply that $N_p/N_{\bar{p}} = 1 + O(10^{-9})$ during baryogenesis
  - The Magnetic-Monopole Problem (Exotic Relics Problem) : the Universe were very hot at early times $\rightarrow$ a large number of heavy, stable magnetic monopoles would be produced, and should be detected observationally
  - Dark Matter
  - Dark Energy

Dark Matter and Dark Energy

- Dark matter and dark energy collectively account for the dark side of the Universe, i.e., the part of the Universe that is not in the luminous (baryonic) form.
- If the standard Big Bang model is the correct gravitational theory of cosmology, then the matter-energy contents of the Universe should comprise the following:
  - $\Omega_m \sim 0.315$, i.e., $\sim 31.5\%$ of the total energy of the Universe exists in the form of non-relativistic matter, of which only a tiny fraction ($\Omega_b \sim 0.049$) is known to exist in baryonic matter form, whereas the remaining ($\sim 85\%$ of) matter is not as yet properly understood and hence is thought to exist in the form of *dark matter*
    - Believed to exist in two forms: non-relativistic Cold Dark Matter ($\Omega_{CDM} \sim 0.268$) and non-relativistic hot dark matter ($\Omega_{HDM} < 0.0152$).
    - Indirect gravitational effects on visible matter, radiation and the large scale structure of the Universe are becoming more and more convincing evidences of its existence.

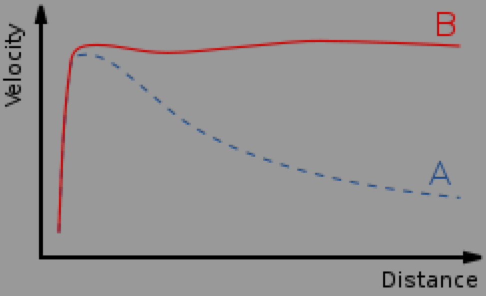

Figure: Rotation curves of galaxies: predicted (A) and observed (B)

- $\Omega_\Lambda \sim 0.685$, i.e., by far the largest portion ($\sim 68.5\%$) is believed to exist in an exotic, unclustered and invisible form of cosmic stuff that permeates all of space, and is known as *dark energy*. Another unknown in the cosmic pie, it was discovered when, in 1998, observational data from supernovae hinted at an accelerating expansion of the present Universe. According to (23) and (18), this would be possible if the Universe is dominated by a component "fluid" with negative pressure. Ironically, this called for the reinstatement of the cosmological constant Λ as the new component fluid with $w_\Lambda = -1$. A cosmological constant dominating at late times can cause cosmic acceleration and make the Universe enter an irreversible de Sitter phase, but this model has two long-standing problems of its own, viz.:
  - the cosmological constant problem: huge ($\sim 10^{120}$ orders of magnitude) discrepancy between the "observed" value of Λ responsible for cosmic acceleration and that predicted by quantum field theoretic arguments for the energy of the quantum vacuum at Planckian scales
  - the coincidence problem: The total fractional energy density is close to 1.0 (or is precisely 1.0 if the Universe is taken to be perfectly flat) at the present time when we are here to observe it, after about 13.82 billion years of expansion when it was always greater than 1.0. Since Ω_Λ is the only constant component, it is natural to be curious about why Λ is so finely tuned as to be dominant only now

- One rather philosophical solution to this is the *Anthropic Principle*: we see the Universe the way it is because we exist
- Dark energy is probably the most extensively speculated candidate in recent years
  - phenomenological, not predicted from the Big Bang cosmology
  - Little understood dynamical nature

Inflation

- Proposed to solve the horizon, flatness, structure and magnetic monopole problems
- Special epoch of exponential expansion moments after the Big Bang ($\sim 10^{-36}s - \sim 10^{-32}s$)
- Decreasing comoving Hubble radius, i.e.,

$$\frac{d(1/aH)}{dt} < 0 \Leftrightarrow \ddot{a} > 0. \quad (69)$$

- Hubble radius swallowed by the outpacing accelerated growth of the expansion
- All physical conditions become correlated on scales much larger than the Hubble radius, thus *smoothing* out the primordial matter fluctuations along the way
- Assumption: at some point in the early universe, the matter energy density was dominated by some form of matter, called a *scalar field* ϕ with a negative pressure
- In the absence of the cosmological constant, the Raychaudhuri equation (23) reduces to

$$\ddot{a} = -\frac{1}{6}(\mu + 3p)a \implies p < -\frac{1}{3}\mu \quad (70)$$

- The Friedmann equation (18) reads

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3}\mu - \frac{k}{a^2}. \quad (71)$$

- Curvature term becomes negligible since the scale factor must increase faster than $a(t) \propto t$

- Exponential growth for the scale factor

$$a(t) \propto e^{Ht} \tag{72}$$

- If the Universe followed the standard radiation-dominated epoch after inflation, then

$$a(t) \propto t^{\frac{1}{2}} \propto \frac{1}{T}, \tag{73}$$

where T is a measure of the typical particle energy: $E \sim k_B T$, $T_0 \sim 3$K and $t_0 \sim \frac{1}{H_0} \sim 10^{18}$s

- Extreme fine-tuning:

$$\frac{a_{inf}}{a_0} \sim \left(\frac{t_{inf}}{t_0}\right)^{\frac{1}{2}} \sim \frac{T_0}{T_{inf}} \tag{74}$$

$$\frac{\Omega_{k,inf}}{\Omega_{k0}} = \left(\frac{H_0}{H_{inf}}\right)^2 \left(\frac{a_0}{a_{inf}}\right)^2 \sim \frac{t_{inf}}{t_0} \sim 10^{-60} - 10^{-54} \tag{75}$$

Standard (Hot Big Bang) Model+Inflation $\rightarrow$ Concordance Cosmology

Inhomogeneous cosmologies

- The Concordance Model (Λ*CDM Cosmology*) is based on large-scale homegeneity and isotropy
- Largely motivated by the observed near isotropy of the CMB and the Cosmological/Copernican Principle
- FLRW geometry may not be the right geometry for all scales
- If the Universe is inherently inhomogeneous, only inhomogeneous cosmological models, such as the Lemaître-Tolman-Bondi (LTB), Szekeres and Kantowski-Sachs models can best describe it

Modified/Generalized/Alternative Theories of Gravity

- Theories of Gravity with Extra Fields
  - Scalar-Tensor theories
    - * Brans-Dicke theories (BD)
  - Einstein-Æther theories
    - * Modified Newtonian Dynamics (MOND)
  - Bimetric theories
  - Tensor-Vector-Scalar theories (TeVeS).
- Higher-dimensional Theories of Gravity
  - Kaluza-Klein (KK) theories
  - Braneworld models
  - Randall-Sundrum (RS) models
  - Dvali-Gabadadze-Porrati (DGP) gravity
  - (Einstein)Gauß-Bonnet (GB) gravity.
- Higher-derivative Theories of Gravity
  - Theories with Ricci and Riemann curvatures in the action
  - Hořava-Lifshitz gravity
  - Galileons
  - $f(R)$ theories: theories with a generic function of the Ricci scalar in the action Lagrangian.

Table: Summary of Gravitaional Models and their Lagrangians:

Model	Action	Matter Lagrangian
GR	$\frac{1}{2}\int d^4x\sqrt{-g}\,(R-2\Lambda)$	$\mathcal{L}_m(g_{ab},\psi)$
BD	$\frac{1}{2}\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega\partial_a\phi\partial^a\phi}{\phi}\right)$	$\mathcal{L}_m(g_{ab},\psi)$
MOND	$\frac{1}{2}\int d^4x\sqrt{-g}\,(R+2\mathcal{L}(g^{ab},A^b))$	$\mathcal{L}_m(g^{ab},\psi)$
TeVeS	$\frac{1}{2}\int d^4x\,(\mathcal{L}_g+\mathcal{L}_s+\mathcal{L}_v)$	$\mathcal{L}_m(g_{ab},\psi)$
$f(R)$	$\frac{1}{2}\int d^4x\sqrt{-g}f(R)$	$\begin{cases}\mathcal{L}_m(g_{ab},\psi) & \text{metric}\\ \mathcal{L}_m(g_{ab},\psi) & \text{Palatini}\\ \mathcal{L}_m(g_{ab},\Gamma^a{}_{bc},\psi) & \text{metric-affine}\end{cases}$
DGP	$\frac{1}{2\kappa_{(5)}}\int d^4x\sqrt{-\tilde{g}}\tilde{R}+\frac{1}{2}\int d^4x\sqrt{-g}R$	$\mathcal{L}_m^{brane}(g_{ab},\psi)$
GB	$\int d^Dx\sqrt{-g}\,(R^2-4R^{ab}R_{ab}+R^{abcd}R_{abcd})$	$\mathcal{L}_m(g_{ab},\psi)$
Galileons	$\frac{1}{2}\int d^4x\sqrt{-g}\left(R+\sum_i^5 c_i\mathcal{L}_i\right)$	$\mathcal{L}_m(f(\pi)g_{ab},\psi_i)$
HL	$\frac{1}{2}\int dtd^Dx\sqrt{-g}\left((\dot{\Phi})^2-\frac{1}{4}\Phi(\nabla^2)^2\Phi\right)$	$\mathcal{L}_m(g_{ij},N,N_i,\psi)$
RS	$\frac{1}{2\kappa_{(5)}}\int d^5x\sqrt{-g_{(5)}}\,(R-2\Lambda_{(5)})-\int d^4x\sqrt{-g}\sigma$	
KK	$\frac{1}{2\kappa_D}\int d^Dx\sqrt{-g_D}R_D$	$\mathcal{L}_m(g_{ab},\psi)$

where ψ is a matter field,Φ is an exotic scalar field, ω is the Dicke coupling constant, A^a is a spacetime 4-vector field, $N^i(x,t)$ is a shift vector, $N(t)$ is a homogeneous lapse function, π is a galleon field, σ is brane tension. $\mathcal{L}_g$, $\mathcal{L}_s$ and $\mathcal{L}_g$ are the actions for the metric (tensor), scalar and vector fields, respectively. κ_D is the bare gravitational constant in D-diemsnions. $\tilde{g}$ and $\tilde{R}$ represent the determinant of the spacetime metric in D-dimensions and its corresponding Ricci scalar.