

# Variable link equation parameters and expected photon returns for the HartRAO Lunar Laser Ranger

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The HartRAO Lunar Laser Ranger (LLR) system requires a state-of-the-art software tool that enables optimal efficiency and signal path parameter estimation. The existing link budget equation estimates the number of returned photons for given conditions and LLR system parameters. This equation is one of the essential mathematical tools that can be considered when developing an integrated system and model for the LLR. The mathematical tool, still under development at HartRAO, can be used to estimate and visualize the relationship between the returned number of photons (observed and computed) and the varying link budget equation parameters. In this work, it is used to indicate “worse and best” parameter values which influence the return signal, presented as an estimate of expected number of returned photons for the HartRAO station. This is all done to achieve optimal efficiency in the LLR signal path in order to yield an improvement in the return-energy of the laser so that ranges to the corner cube retro-reflectors can be measured accurately. The geographic position of the HartRAO station, new state-of-the-art HartRAO LLR system under development and the expected number of returned photons will enable HartRAO to play a key role in improving the ranging accuracy to a sub-centimetre level. This will enhance the current effort to determine highly accurate Earth-Moon distances for various scientific purposes.

## Introduction

The existing link budget equation estimates the number of returned photons for given conditions and LLR system parameters. It calculates the mean number of returned photons counted by a photon detector as illustrated in (Degnan 1993), as

$$n_p = \eta_q \left( E_T \frac{\lambda}{hc} \right) \eta_t G_t \sigma \left( \frac{1}{4\pi R^2} \right)^2 A_r \eta_r T_a^2 T_c^2, \quad (1)$$

where  $\eta_q$  is the quantum detector efficiency,  $E_T$  the total laser pulse energy,  $\eta_t$  the transmit optics efficiency,  $G_t$  the transmitter gain,  $\sigma$  the satellite/Moon reflector optical cross-sectional area,  $R$  the slant range,  $A_r$  the effective area of the receiving telescope aperture,  $\eta_r$  the receive optics efficiency,  $T_a$  is the one-way atmospheric transmission and  $T_c$  is the cirrus cloud cover.

In this work, we focus on the effects of the three variable link budget parameters on the expected number of returned photons for the HartRAO LLR system represented as,

$$n_p = C_{system} \left( \frac{T_a T_c}{R^2} \right)^2, \quad C_{system} = \eta_q \left( E_T \frac{\lambda}{hc} \right) \eta_t G_t \sigma \left( \frac{1}{4\pi} \right)^2 A_r \eta_r \quad (2)$$

where  $C_{system}$  is the constant of the “fixed” system parameters as described in Equation 1. These effects are studied properly to achieve optimal efficiency in the HartRAO state-of-the-art LLR signal path. This will yield an improvement in the return-energy of the laser, hence more data quantity and quality will be achieved from the only (currently) LLR station in the Southern Hemisphere (Combrinck and Botha, 2013).

## Methodology

Thermal and density fluctuations are major limiting factors whenever a laser beam propagates through the atmosphere and was recently investigated further by our group (Ndlovu and Chetty 2014). Other detrimental effects on the propagating laser beam are introduced by the signal path, transmitting and receiving optics and the detection system (see Figures 1 to 4). This causes the laser to propagate through different media and thus, introduces time delays that can affect the range measured.

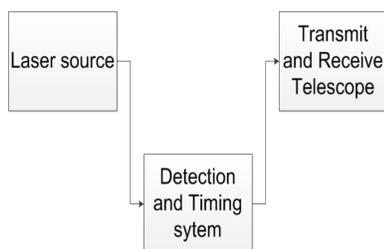


Figure 1: Basic description of the HartRAO LLR laser timing signals for emitted pulse and receive window.



Figure 2: The ex-OCA 1 m telescope installed on its foundation at HartRAO.



Figure 3: The old AC motor system for controlling the beam expander.



Figure 4: The new DC motor to control the objective lens of the beam expander.

## Results, Analysis and Discussion

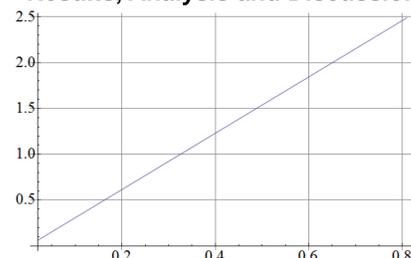


Figure 5: Received photons per minute vs. varying atmospheric transmission.

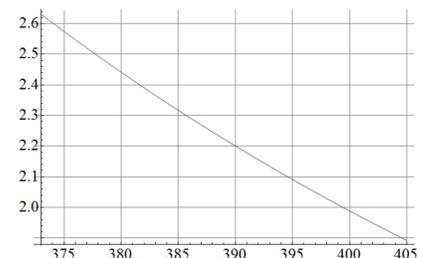


Figure 6: Received photons per minute vs. the slant range ( $\times 10^3$  km).

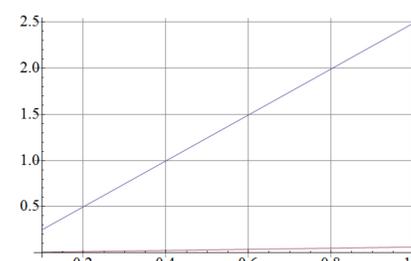


Figure 7: Received photons per minute vs. cirrus cloud transmission, for atm. transmission at min. (red) and max. (blue).

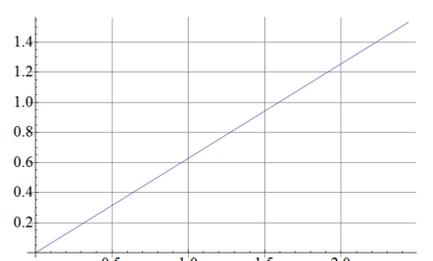


Figure 8: Received photons per minute vs. fixed system parameters ( $C_{system} \times 10^{35}$ ).

The graphs (Figures 5 to 8) indicate the relationship between the atmospheric fluctuations, slant range and the number received photons. The slant range  $R$  is one of the biggest influences on the number of received photons (Figure 6). Taking the minimum atmospheric transmission to be 0.02 and its maximum being 0.81 (refer to Figure 7), the maximum number of returned photons depends on “better” atmospheric conditions (Table 1).

Table 1: The relationship between varying parameters and number of photons.

Parameter	Worst value	Optimal value
Laser pulse energy (mJ)	100	110
Transmit optics efficiency	0.4	0.9
Slant range (km)	405000	378084
Detector quantum eff.	0.4	0.7
Receive optics eff.	0.4	0.9
Atmospheric transmission	0.02	0.81
Cirrus transmission	0.1	1
Returned photons/minute	0.003	15

## Conclusion

In conclusion, expansion of the existing link equation is required in order to consider those time delays (that affects the range) and other factors, which can be caused by the detection system as the photons are transmitted and received through the LLR signal path. This will help improve the system signal-to-noise ratio, thus ensuring more photon returns for the HartRAO LLR.

## Acknowledgements

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## References

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