Correlation between mathematics proficiency and performance in a first-year physics course

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Abstract. We report here on the results of a mathematics entry test given to the 2013 intake of first year students enrolled for first year introductory physics course (non-major) in the faculties of Science and Health Sciences at the University of Johannesburg. The test was designed to assess the background knowledge and mathematical skills deemed to be necessary for the students to successfully undertake an algebra-based first year physics course. The topics covered in the test were based on the grade 11 and grade 12 mathematics syllabi and the use of calculators was prohibited. On comparison of the results of this test with the matric marks for mathematics, it has become apparent that the latter is inflated and reflects neither the real skills nor the success rate of students at university level. This is reflected on the macroscopic level in the comparison between the class average of the test (42%) and the class average of the matric mathematics marks (65%). The mathematics entry test also revealed the inability of students to solve simple numerical calculations, inability that can be attributed to the use of calculators from an early age. A strong correlation between the lack of basic mathematics skills and the performance of students in our first year physics course has been found. Finally, Grade 12 physical science marks, on which the admission to university is based, are proved to be inflated when compared with the performance of students in first year physics.

1. Introduction

Mathematics is the language of physics and is aptly chosen for its simplicity and accuracy. The association between mathematics and physics is embedded in the minds of science students from a young age. Counting was one of the first scientific inventions of primitive man [1]. Physics is quite simply the study of physical nature and mathematics is one of the oldest scientific tools and so there is an inherent relation between the two. The laws, theories and principles of physics translate into equations, the resolution of which resides in mathematics. According to Gross [2], mathematics and physics have had the longest union and the most fruitful exchanges.

The study of mathematical transition from the secondary to the tertiary level of education is reflected in recent literature [3, 4]. Questionnaires and quizzes are often a means to gauge the students preparedness for tertiary study. Jennings [3] attributed the decline in mathematical proficiency of students to two main factors - altering entry requirements of universities by dropping prerequisites to attract more students and the diverse cultural and educational backgrounds that the students have. Regardless of the cause, the effects of this are far reaching. Mathematical aptitude is a non-negotiable pre-requisite for problem solving in physics. Redish [5] explains how to use mathematics to describe a physical system related to problem solving. Firstly, in tackling the problem or describing the physical system, the ability *to get the physics right* is crucial. Students must be able to decide and focus on just the vital parts of the information given and discard the rest. The next step is creating a mathematical model. This requires knowledge of the mathematical operations and structures that are available and applicable to the problem at hand. Once the mathematical model is set up, it needs to be processed: in other words, the equation needs to be solved. This is followed by interpretation and evaluation. The last two steps are perhaps the most crucial as students need to be able to judge whether their results describe the physical system or whether their mathematical modelling step needs to be revisited.

The auxiliary physics course at the University of Johannesburg is offered in the first year of the three and four year bachelors degrees of life sciences and health sciences, respectively. This course is algebra based and requires an average of at least 50% in matric mathematics and physical sciences (PS). Despite having marks in mathematics and PS well above the required minimum (the average matric mathematics and PS marks for this group are 65% and 66%, respectively), the performance of the students in the mathematics entry test and the subsequent physics tests has been poor. This paper aims to suggest a correlation between mathematical aptitude and performance in physics.

2. Method

We have compiled the results of a mathematics entry test that was given to 103 first year students of the 2013 intake in an introductory physics course (non-major) in the Faculties of Science and Health Sciences at the University of Johannesburg. The two authors of this paper are the lecturers of this course. Here we report and analyse the results only for those students who wrote the National Senior Certificate examinations at the end of 2012, and enrolled at University at the beginning of 2013 (103, as stated above). Such students have therefore attempted to pass physics for the first time in 2013 (i.e. repeating students and senior students are excluded from this analysis). The test was given to the students at the beginning of the second week of the first term. This was deliberately done in order to discard the influence of notions acquired at the university on the results of the study. The topics of the test were communicated to the students during the first lecture of term, i.e. six days before the test was written. The test was set out of 86 marks, it had a duration of 1 hour and 50 minutes and the use of calculators was prohibited. The test was constituted by six sets of questions based on the current South African grade 11 and grade 12 syllabi. In particular:

- Question 2 (17 marks): factorization and simplification of fractions (hereafter Q2);
- Question 3 (11 marks): addition of fractions (Q3);
- Question 4 (9 marks): exponents (zero and negative exponents, fractional exponents) (Q4);
- Question 5 (29 marks): linear, quadratic and logarithmic equations (Q5);
- Question 6 (14 marks): simultaneous solving of sets of two quadratic equations (Q6).

Problems used to compile the test were taken from the textbooks in Refs. [6, 7]. Moreover, students were asked to solve six simple calculations (including powers and roots) without a calculator (question 1 for 6 marks, hereafter Q1). Given the relevance of this particular question in the following part of the paper, we considered worthwhile to reproduce Q1 in detail [8]: a) 8×17; b) $(10^7)^2$; c) $(0.2)^3$; d) $\sqrt{0.01}$; e) $25 \times 13 + 13^2$; f) $2^6 + 7 - 36 \times 24$.

The mathematics test marks (MTM) were thoroughly analysed against the performance of the students in the theory part of the course at the end of the semester, i.e. end of May 2013, reflected in the theory mark (TM) (calculated considering 80% from two semester tests and the rest from minor assessments, i.e. discarding the performance in the practical part of the course). Many studies have in fact shown how the proficiency in mathematics is positively correlated to their performance in university first year physics courses [9]. In particular, the linear correlation between the mathematics test marks (X) and the theory marks (Y) was evaluated by means of the Pearson product-moment correlation coefficient r [10], defined as

$$r = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}} \tag{1}$$

where (X_i, Y_i) represent the paired data for each student, $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ represent the mean values of X and Y respectively, and $n = 103$ in the present study. The Pearson coefficient is commonly used in statistical analysis in order to measure the linear correlation between two sets of data.

3. Results and Discussion

Table 1 reports the analysis of the performance of the students for the overall test and for each of the six questions. In particular, the average, the lowest and highest marks are reported together with the Pearson coefficient r calculated against the TM. A class average of 42% reveals an overall poor performance of the students in this assessment. Interestingly, the mathematics test marks has a good degree of correlation with the TM, represented by a coefficient r of 0.47. Such correlation can be seen in the plot of the TM versus the mathematics test marks in Figure 1 (a). As a minimum TM of 40% is required in order for students to write the examination, the

	Class Average Mark	Lowest Mark	Highest mark	r
Test	42%	12%	74%	**0.47**
Q1	2.6 marks	0 marks	6 marks	**0.48**
Q2	36%	0%	82%	0.37
Q3	35%	0%	100%	0.26
Q4	32%	0%	100%	0.09
Q5	41%	14%	83%	0.34
Q6	61%	0%	100%	0.34

Table 1. Average mark, lowest mark, highest mark and Pearson coefficient r (measured against the semester mark of the students) for the overall mathematics test and for each of the six individual questions.

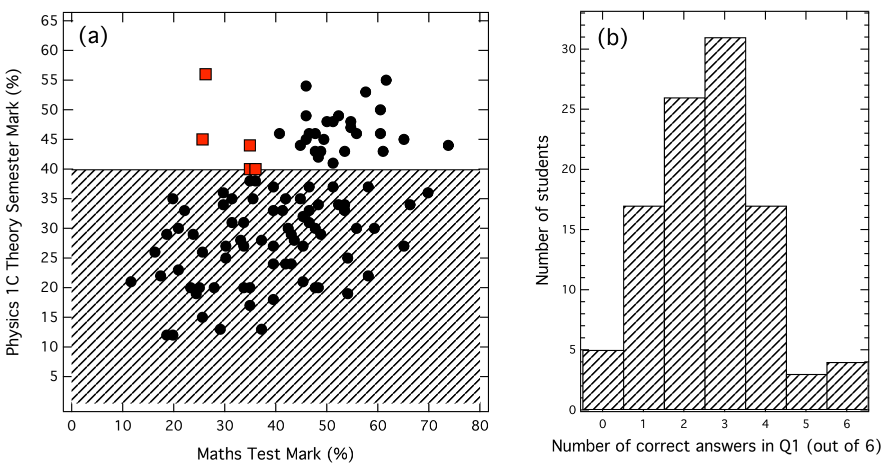

Figure 1. (a) Plot of the theory marks versus the mathematics marks for Physics 1C students. (b) Histogram of the performance of the students in Q1.

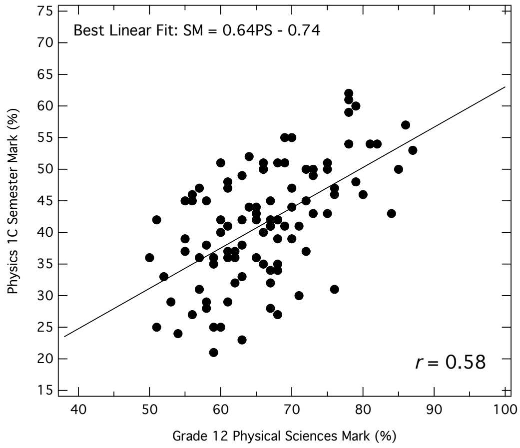

Figure 2. Plot of the Physics 1C semester marks (SM) versus the Grade 12 PS marks. The straight line represents the best fit to the data.

shaded region in the plot serves to highlight those students who did not gain exam entrance. It can therefore be seen that, apart from 5 students highlighted with red squares, no student with a maths test mark below 40% achieved a physics SM of 40%, i.e. had a performance during the semester enough to grant him/her access to examination. This indicates that this test can be used to predict, to a certain extent, the success of students in this physics course. The above seems to be a necessary condition, but not a sufficient one, as some of the students with marks above 40% in the maths test still did not qualify for the examination.

Looking at the performance in each question, it is interesting to note that the r coefficients for Q2 to Q6 are low, showing a poor correlation between the performance in these questions and the TM. In particular, the coefficient r for Q4 is very close to zero. This is attributed to the fact that this question assessed the background knowledge in exponents, a less relevant topic as far as problem solving of algebra-based physics problems is concerned. On the other side, the correlation between the marks scored in Q1 and the TM is higher than the one for the other questions, and it is as good as the correlation of the maths test itself ($r = 0.48$). This result is particularly interesting if we consider that this question was meant to assess the capability to perform simple calculations without the use of a calculator, and the background knowledge required to do so is meant to be acquired long before Grades 11 and 12. This finding suggests that the assessment of such basic skills is as good to predict the students' performance in first year physics as the assessment of the background knowledge of those topics widely used in solving problems in an algebra-based physics course, such as quadratic equations and the algebra of fractions. Taken together, the fact that Q2 to Q6 show poor correlation with the physics performance seems to indicate that students are taught to memorise solving methods without the understanding of the basic rules. This is reflected in the struggle to apply the same rules when different symbols are used, i.e. physical symbols instead of the habitual x, y, z or a, b, c.

It is worth noting that, as reported in the histogram in Figure 1 (b), only 4 students out of 103 were able to score full marks for Q1. This is particularly worrying when we consider that the typical mistakes derived from the lack of knowledge of multiplication and division priorities in solving the numerical expressions, lack of knowledge of the rules in exponents and above all the lack of knowledge of the meaning of a square root and powers. Below are examples of the common misconceptions we dealt with this year:

Example 1: $\sqrt{v_f^2 - v_i^2} = v_f - v_i$

Example 2: $\frac{v_i^2+v_f^2}{v_i+v_f} = v_i + v_f$

Example 3: $F = mg\sin\theta - \mu_k mg\cos\theta$, giving - after dividing by μ_k on both sides: $F = mg\frac{\sin\theta}{\mu_k} - mg\cos\theta$

Finally, we have analysed the correlation between the Physics 1C final semester marks (calculated by adding 70% of the TM to 30% of the final practical mark) and the Grade 12 marks in PS, as shown in Figure 2. The Pearson correlation coefficient between these two sets of data is good ($r = 0.58$), showing that the matric marks can be used to predict students performance in first year physics, with a degree of uncertainty given by the fact that our data show that school marks have been considerablly inflated. In fact, the best linear fit to the data in Figure 2 is described by the following equation: $SM = 0.64PS - 0.74$, where SM represents the first year physics semester mark and PS the Grade 12 PS mark.

4. Concluding remarks

Our rigorous analysis has led us to conclude that the underlying problem leading to pitiable throughput is poor understanding in basic mathematics. Solving physics problems, a bulk portion of all physics assessments, is close to impossible with a shaky foundation in mathematics and will lead to an undesired collapse.

Our results suggest that, in order to improve the course throughput in first year physics courses, universities should not base students admission uniquely on Grade 12 marks, but also on admission tests aimed at assessing the real background knowledge of matriculants. Alternatively, any interventions implemented at the university level should firstly concentrate on the basic mathematical skills discussed in this paper, rather than on physical principles.

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