Nonlocality arguments in the temporal Clauser-Horne-Shimony-Holt scenario

Sujit K Choudhary, Sandeep K Goyal, Thomas Konrad

School of Chemistry and Physics, University of KwaZulu-Natal, Durban 4000, South Africa.

E-mail: `choudhary@ukzn.ac.za, goyal@ukzn.ac.za, konradt@ukzn.ac.za`

Abstract. For testing the existence of superposition of macroscopically distinct quantum states, Leggett and Garg [Leggett A J, Garg A 1985 Phys. Rev. Lett. **54** 857], proposed an inequality based on the assumptions of 'macroscopic realism', 'noninvasive measurabilty' and 'induction'. These assumptions in a slightly different form have been used to derive a temporal version of Bell's inequality [Brukner C, Taylor S, Cheung S, Vedral V (e-print quant-ph/0402127)]. This inequality is violated in Quantum Mechanics and thus establishes non-locality of quantum correlations in time. We compare various nonlocality arguments in this temporal scenario.

1. Introduction

Quantum mechanics is a mathematical theory for describing the physical world but it is probabilistic in nature. This may not be surprising, however. What is surprising is that (according to the Copenhagen interpretation) this probability is not the probability of some dynamical variable having a particular value in some state, rather it represents the probability of finding a particular value of the variable if that dynamical variable is measured. So what about the variables when the system is not subjected to any measurement; quantum mechanics remains mute in this regard. This interpretation generated numerous debates among physicists which resulted into the development of Hidden Variable Theories. The aim of hidden variable theories is to provide a formalism which, while being empirically equivalent to quantum mechanics, does not contain the intrinsic indeterminacy of quantum mechanics *i.e.*, in this theory quantum probabilities become epistemic, they arise due to our ignorance about hidden variables whose knowledge would give us the precise value of every observable.

In a much celebrated paper [1], J.S. Bell showed that this realistic viewpoint cannot explain some correlations predicted by quantum mechanics unless it assumes some signalling between the correlated events. Such correlations are called nonlocal. Bell showed this by means of an inequality, known as Bell's inequality [1]. Later, Hardy [2] gave an argument which also reveals the nonlocal character of Quantum Mechanics, but his argument, unlike Bell's argument, does not use inequalities involving expectation values. Afterwards, Cabello [3] introduced another logical structure to prove Bell's theorem, namely predictions of quantum mechanics are not compatible with the notion of local-realism, without inequality. Although, Cabello's logical structure was originally proposed for establishing nonlocality for three particle states, it was later exploited to establish nonlocality for a class of two-qubit mixed entangled state [4]. It is noteworthy here that in contrast, there is no two-qubit mixed state which shows Hardy type nonlocality [5] whereas almost all pure entangled states of two-qubits do so (maximally entangled states are the exception) [6, 7]. Likewise, for almost all two-qubit pure entangled states other than maximally entangled ones, Cabello's nonlocality argument works and for these states, the maximum probability of success of this argument is 0.11 [8]. This is interesting, as for two-qubit states, the maximum success probability of Hardy's argument is known to be 0.09 [6, 9].

Although quantum theory predicts the existence of nonlocal correlations but these correlations cannot be exploited to communicate with a speed greater than that of the light in vacuum. But quantum theory is not the only nonlocal theory consistent with the relativistic causality [10]. Theories which predict nonlocal correlations and hence permit violation of Bells inequality but are constrained with the no signalling condition are called Generalized Nonlocal Theory (GNLT). Success probabilities of Hardy's and Cabello type arguments have also been compared in the framework of GNLT [11]. It has been shown there that for two two-level systems, success probabilities of both these arguments converge to a common maximum, 0.5.

Recently, the principle of nonviolation of information causality [12] has been proposed as one of the foundational principles of nature. Hardy and Cabello arguments have also been studied in the context of the above principle. Once again for two two-level systems, the success probabilities of these arguments converge to a common maximum, 0.20717 [13].

For testing the existence of superposition of macroscopically distinct quantum states, Leggett and Garg [14], proposed an inequality based on the assumptions of macroscopic realism, noninvasive measurabilty and induction. These assumptions in a slightly different form have been used to derive a temporal version of Bell's inequalities [15]. They are the constraints on certain combinations of temporal correlations for measurements of a single quantum system, which are performed at different times. These inequalities are violated in Quantum Mechanics and thereby give rise to the notion of *entanglement in time*. Recently, Hardy's argument has been studied in the temporal Bell-CHSH scenario [16]. It has been shown there that the maximum probability of success of this argument can assume up to 25%. We describe Cabello's argument in this temporal scenario as the probability of its success in revealing the time-nonlocal features of quantum states can be more than that of the Hardy's argument.

2. Temporal Bell-CHSH inequality

The temporal Bell inequalities are derived from the following two assumptions [15]

(i) *Realism*: The measurement results are determined by hidden properties, which the particles carry prior to and independent of observation, and

(ii) *Locality in time*: The result of a measurement performed at time t_2 is independent of any measurement performed at some earlier or later time t_1.

These assumptions are similar to the assumptions made by Leggett and Garg [14] in the context of testing superpositions of macroscopically distinct quantum states. But here these assumptions are more general in the sense that they do not necessarily demand the physical system under consideration to be macroscopic [17].

In the framework of a probabilistic theory, consider a physical system on which one of the two observers, Alice conducts the experiments of measuring any one (chosen at random) of the two $\{-1, +1\}$-valued random variables a_1 and a_2 whereas another observer Bob can run the experiments of measuring any one (chosen at random) of the two $\{-1, +1\}$-valued random variables b_1 and b_2 at a later time (this we call the temporal CHSH scenario).

Consider now the quantity B defined as:

$$B = a_1[b_1 + b_2] + a_2[b_1 - b_2].$$

In a realistic theory the value of the quantity B cannot be other than 2 or -2 if the theory is also local in time. After averaging this expression over many runs of the sequence of measurements, one obtains

$$-2 \leq \langle B \rangle \leq 2 \tag{1}$$

i.e.

$$|\langle a_1 b_1 \rangle + \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle - \langle a_2 b_2 \rangle| \leq 2. \tag{2}$$

This we call temporal Bell-CHSH inequality in analogy to the spatial one. As shown in [15], this inequality is violated in quantum mechanics if the system under consideration is a single spin-1/2 particle and observables a_1, a_2, b_1 and b_2 are the spin observables in directions given by the unit vectors $\tilde{a_1}$, $\tilde{a_2}$, $\tilde{b_1}$ and $\tilde{b_2}$ respectively where the spin directions are related as $\tilde{a_1} = \frac{1}{\sqrt{2}}(\tilde{b_1} + \tilde{b_2})$ and $\tilde{a_2} = \frac{1}{\sqrt{2}}(\tilde{b_1} - \tilde{b_2})$. In fact, for these observable settings the violation is maximum and it is equal to $2\sqrt{2}$. This may be called the temporal Tsirelson's bound [15, 18].

3. Temporal version of Hardy's argument

Consider the following four conditions in the above mentioned temporal CHSH scenario:

$$\left.\begin{array}{rcl} \text{Prob}(a_1 = +1,\ b_1 = +1) & = & 0, \\ \text{Prob}(a_2 = +1,\ b_1 = -1) & = & 0, \\ \text{Prob}(a_1 = -1,\ b_2 = +1) & = & 0, \\ \text{Prob}(a_2 = +1,\ b_2 = +1) & = & q \end{array}\right\} \text{(with } q > 0). \tag{3}$$

The above four conditions together form the basis of Hardy's nonlocality argument. The last equation, for example, states that if Alice measures a_2 and Bob measures b_2 at some later time, then the probability that both will get $+1$ as their measurement results is q. These four conditions cannot be fulfilled simultaneously in a theory which is realistic and time-local [16, 19].

On the contrary, a qubit prepared in the state $|x+\rangle$ with the measurement setting $a_1 = -\sigma_x, a_2 = \sigma_y, b_1 = \sigma_y, b_2 = -\sigma_x$ satisfies all these conditions with $\text{Prob}(a_2 = 1, b_2 = 1) = 0.25$ which indeed is its optimal value [16]. Thus the temporal version of Hardy's proof is considerably stronger than its spatial analog where $\text{Prob}(a_2 = 1, b_2 = 1)$ can be no greater than 0.09. This fact has been recently verified in an experiment [19].

4. Cabello's argument in the temporal scenario

Cabello's conditions result by replacing the right hand side of the first condition of (3) with a nonzero probability p with $p < q$, and keeping the remaining three conditions the same.

It can easily be seen that these equations contradict local-realism if $0 \leq p < q$. To see this, let us consider those realistic (hidden variable) states for which $a_2 = +1$ and $b_2 = +1$. For these states, the second and the third equations in (3) tell that the values of a_1 and b_1 must be equal to $+1$. Thus according to the assumptions of *locality in time* and *realism* $\text{Prob}(a_1 = +1,\ b_1 = +1)$ should at least be equal to q. This contradicts the Cabello's argument as $p < q$. It should be noted here that $p = 0$ reduces this argument to Hardy's one. So by Cabello's argument, we specifically mean that the above argument is valid, even for nonzero p. The probability of success of this argument is measured by the difference in the two nonzero probabilities appearing in the argument, i.e. by $q - p$ [8]. As this argument is more relaxed than Hardy's argument, so we expect it to show a higher violation of timelocal-realism of quantum mechanics.

5. Discussion

We discussed various nonlocality arguments in the temporal scenario. In particular, we reviewed the Bell-like inequality, the Hardy and Cabello-like arguments. Hardy's argument has already been tested to be stronger in the temporal situation than its spatial version. As we have seen that Cabello's argument is more relaxed than Hardy's argument, therefore we expect it to show a violation at least equal to Hardy's. In fact, we expect it to show a higher violation of timelocal-realism of quantum mechanics which will be subject of future study.

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