The Equation of State (EoS) of hadronic matter from the microscopic Ultra-relativistic Quantum Molecular Dynamics (UrQMD) model

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Abstract. The Equation of State (EoS) of a hot and dense hadron matter is studied using a microscopic transport model which can support the Large Hadron Collider energies of up to $\sqrt{s_{nn}}$= 14 TeV, namely the Ultra-relativistic Quantum Molecular Dynamics (UrQMD). The molecular dynamics simulation is performed for a system of light meson species (π; ρ and K) in a box with periodic boundary conditions. The equilibrium state is investigated by studying the chemical equilibrium and the thermal equilibrium of the system. The particle multiplicity equilibrates with time, and the energy spectra of different light meson species have the same slopes and common temperatures when thermal equilibrium is reached. The solution of the EoS allows for better understanding of the final state of interactions, which is dominated by hadrons produced during ultra-relativistic heavy ion collisions.

1. Introduction

A large number of studies in heavy ion physics and high energy physics have been done using the results from the Relativistic Heavy Ion Collider (RHIC). Now with the restart of the Large Hadron Collider (LHC) physics programme, the field of high energy nuclear physics, and especially heavy ion physics, has gone into a new era. It is now possible to explore the properties of Quantum-Chromo-Dynamics (QCD) at unprecedented particle densities and temperatures, and at much higher energies than that produced at RHIC, from $\sqrt{s} = 200$ GeV to $\sqrt{s} = 14$ TeV at the LHC [1].

High energy heavy ion reactions are studied experimentally and theoretically to obtain information about the properties of nuclear matter under extreme conditions at high densities and temperatures, as well as about the phase transition to a new state of matter, the quark-gluon plasma (QGP) [2, 3, 4, 5]. This work reports on the equation of state of hadronic matter extracted using the UrQMD model. The knowledge of the equation of state (EoS) is important for better understanding of the final state of interactions which is dominated by hadrons produced during ultra-relativistic heavy ion collisions. The EoS of nuclear matter is one of the key points to gain further understanding since EoS directly provides the relationship between the pressure and the energy at a given net-baryon density [6]. The thermodynamic properties, transport coefficients and EoS for hadron gas are a major source of uncertainties in dissipative fluid dynamics [4, 7, 8].

In order to study the EoS we look at the system in equilibrium. Equilibration of the system is studied by evaluating particle number densities from chemical equilibrium, energy spectra as well as the temperatures from thermal equilibrium of different light meson species in a cubic box, which imposes periodic boundary conditions. The infinite hadronic matter is modelled by initializing the system with light meson species namely, the pion (π), the rho (ρ) and the kaon (K).

We focus on the hadronic scale temperature (100 MeV $< T <$ 200 MeV) and net zero baryon number density, which are expected to be realized in the central high energy nuclear collisions [9]. We then change energy density from ε = 0.1 - 2.0 GeV/fm^3 and for each energy density we run the system with n-number of events while keeping the volume and baryon number density constant until the equilibrium state is reached.

The rest of the paper is organized as follows: In section 2 we study the description of the UrQMD model. In section 3 we study equilibration properties of the system. In section 4 we study the equation of state (EoS) of the hadron matter.

2. Brief Description of the UrQMD Model

The Ultra-relativistic Quantum Molecular Dynamic model (UrQMD) is a microscopic model based on a phase space description of nuclear reactions. We use version 3.3 of the UrQMD model for this study. The UrQMD 3.3 hybrid approach extends previous ansatzes to combine the hydrodynamic and the transport models for the relativistic energies. The combination of these approaches into one single framework, it is done for a consistent description of the dynamics

The UrQMD model describes the phenomenology of hadronic interactions at low and intermediate energies from a few hundreds MeV up to the new LHC energy of $\sqrt{s}$ =14 TeV per nucleon in the centre of mass system [10, 11]. The UrQMD collision term contains 55 different baryon species and 32 meson species, which are supplemented by their corresponding anti-particles and all the isospin-projected states [10, 12]. The properties of the baryons and the baryon-resonances which can be populated in UrQMD can be found in [12], together with their respective mesons and the meson-resonances. A collision between two hadrons will occur if

$$d_{trans} \leq \sqrt{\frac{\sigma_{tot}}{\pi}}, \qquad \sigma_{tot} = \sigma(\sqrt{s}, type), \tag{1}$$

where d_{trans} and σ_{tot} are the impact parameter and the total cross-section of the two hadrons respectively [10]. In the UrQMD model, the total cross-section σ_{tot} depends on the isospins of colliding particles, their flavour and the centre-of-mass (c.m) energy $\sqrt{s}$. More details about the UrQMD model is presented in [10, 11, 12].

3. Equilibration of Hadronic Matter

To investigate the equilibrition of a system, the UrQMD model is used to simulate the ultra-relativistic heavy ion collisions. A multi-particle production plays an important role in the equilibration of the hadronic gas [4]. The cubic box used for this study is initialised according to the following numbers of baryons and mesons: zero protons, 80 pions, 80 rhos and 80 kaons. For this study a cubic box with volume V and a baryon number density n_B = 0.00 fm^{-3} is considered. The energy density ε, volume V and the baryon number density n_B in the box are fixed as input parameters and are conserved throughout the simulation. The energy density is defined as $\varepsilon = \frac{E}{V}$, where E is the energy of N particles and the three-momenta p_i of the particles in the initial state are randomly distributed in the centre of mass system of the particles as shown in the equations below.

$$E = \sum_{i=1}^{N} \sqrt{m_i^2 + p_i^2}, \qquad \sum_{i=1}^{N} p_i = 0. \tag{2}$$

3.1. Chemical Equilibrium

In this subsection the chemical equilibrium is studied from the particle number densities of different light meson species in a box with $V = 1000$ fm^3, zero net baryon number density $n_B = 0.0$ fm^{-3} at different energy densities using UrQMD box calculations. Figure 1 (a) and (b) represents the time evolutions of the various meson number densities at $\varepsilon = 0.3$ and 0.4 GeV/fm^3 energy densities.

In figures 1 (a) and (b), the meson species indicate that the system does indeed reach chemical equilibrium. It is observed that the pions have large particle number densities and the reason could be the decay in the heavier mesons and other particles produced in the system to form pions. The saturation of the particle number densities indicate the realization of a local equilibrium. In conclusion, the chemical equilibrium of the system has been reached, as in both figures the saturation times are the same for all three mesons, regardless of the shape of each meson. In figure 1 (a) where $\varepsilon = 0.3$ GeV/fm^3, the equilibrium time for all meson species is around $t = 22$ fm/c and for figure 1 (b) at a higher energy density of $\varepsilon = 0.4$ GeV/fm^3, the equilibrium time is observed to have increased to $t = 32$ fm/c. These results show that an increase in energy density influences the particle multiplicity inside the periodic box, which affect the equilibration time.

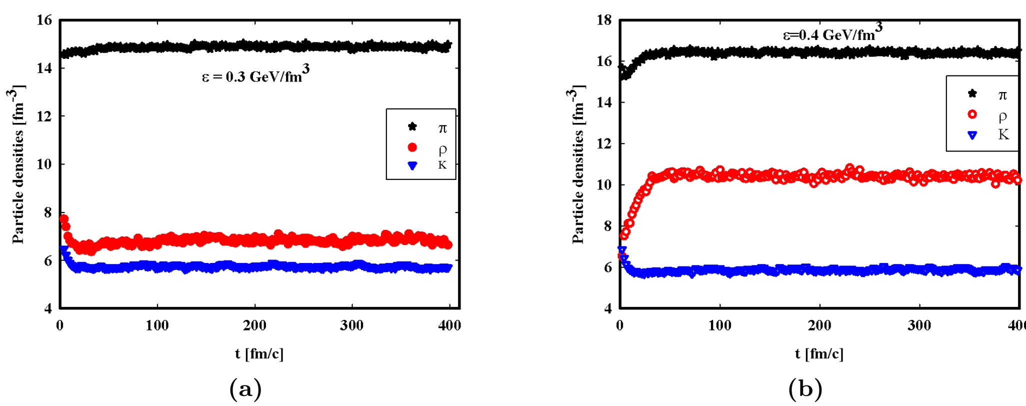

Figure 1: The time evolution of particle number densities of light meson species (π, ρ and K) at (a) $\varepsilon = 0.3$ GeV/fm^3 and (b) $\varepsilon = 0.4$ GeV/fm^3.

3.2. Thermal Equilibrium and Temperature

In this subsection the thermal equilibrium and the temperature from the energy distributions of different light meson species are studied. The possibility of the thermal equilibrium of the hadronic matter is studied by examining the energy distribution of the system in a box with periodic boundary conditions using the UrQMD model. The particle spectra are given by the momentum distribution as [13]

$$\frac{dN_i}{d^3p} = \frac{dN}{4\pi E p dE} \propto C e^{(-\beta E_i)}. \tag{3}$$

Figure 2 (a) and (b) represent the time evolutions of energy spectra of different meson species. The linear lines are fitted using the Boltzmann distribution, which is aproximated by $C\exp(-\beta E_i)$ from Eq. 3, where $\beta = 1/T$ is the slope parameter of the distribution and E_i is the energy of particle i. The results are plotted as a function of kinetic energy $K = E - m$, so that the horizontal axes for all the particle species coincide [14]. In figure 2 (a) and (b) it is observed that the slopes of the energy distribution converge to common values of temperatures at different times above t = 180 fm/c for ε = 0.2 GeV/fm^3 and above t = 250 fm/c for ε = 0.3 GeV/fm^3. In thermal equilibrium the system is characterized by unique temeperature T [14]. The thermal temperatures were extracted from the equilibriumn state using the Boltzmann distribution such that T = 118.3 MeV for ε = 0.2 GeV/fm^3 and T = 150.1 MeV for ε = 0.3 GeV/fm^3.

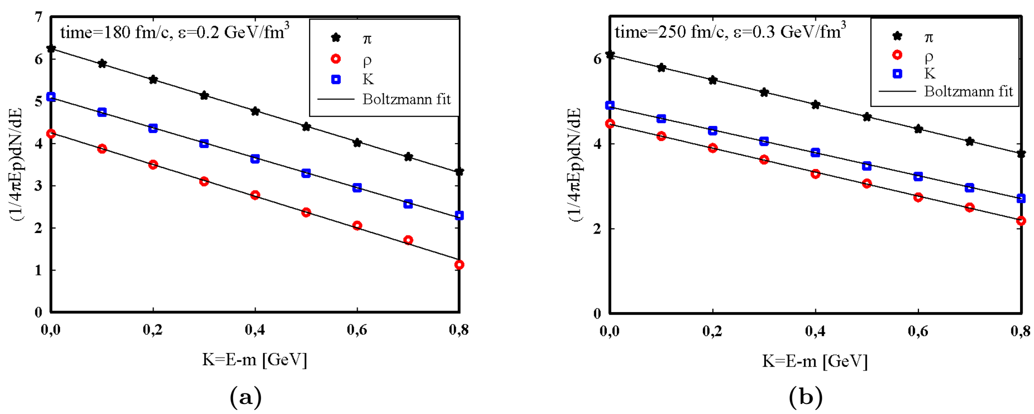

Figure 2: The energy distributions of light meson species (π, ρ and K) at (a) ε = 0.2 GeV/fm^3 and t = 180 fm/c and (b) ε = 0.3 GeV/fm^3 and t = 250 fm/c. The lines are the Boltzmann fit which gives the extracted temperatures of T = 118.3 MeV for (a) and T = 150.1 MeV for (b).

4. Hadronic gas model (Equation of State)

The hadron abundances and the ratios have been suggested as the possible signatures for the exotic states and the phase transitions of the nuclear matter [15]. The equation of state of hot and dense hadron matter provides the valuable information regarding the nature of the hadron matter. These signatures have been applied to the study of chemical equilibration in the relativistic heavy ion reactions. The properties like the temperatures, the entropies and chemical potentials of the hadronic matter have been extracted assuming thermal and chemical equilibrium [10].

In this section the EoS for a hadron matter is studied from the UrQMD simulation. The equation of state can be studied using a statistical model which is described by the grand canonical ensemble of non-interacting hadrons in an equilibrium sate at temperature T as presented in [4, 12]. A large number of studies have been done to study EoS of hadron matter [4, 12, 16, 17]. For this study the focus is only on the EoS of hadrong matter made out of the π, the ρ and the K calculated from the UrQMD model. This is done through studying the evolution of pressure and energy density with temperature. The thermodynamic quantities used to calculate EoS are the energy density given by

$$\varepsilon = \frac{1}{V}\sum_{i=1}^{all-particles} E_i \,, \tag{4}$$

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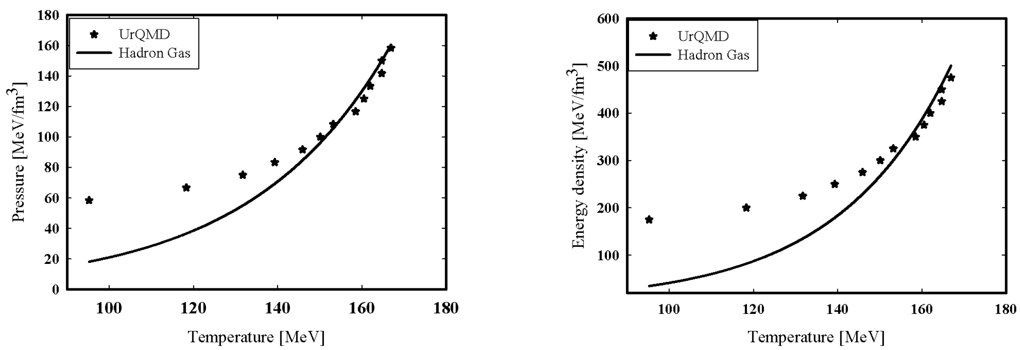
and the pressure which is given by

$$P = \frac{1}{3V} \sum_{i=1}^{all-particles} \frac{p_i^2}{E_i}. \tag{5}$$

Figure 3 represent the EoS of hadronic matter namely (a) the pressure and, (b) the energy density as function of temperature. The energy density used here Eq. (4) is the same energy density used as an input parameter during the initialization of the simulation. The pressure is then calculated from the collision results using Eq. (5). The thermal temperatures used here are extracted from the thermal equilibrium (subsection 3.2) using the Boltzmann distribution function given as $C\exp(\frac{E}{T})$ [18].



(a) (b)

Figure 3: The equation of state of a mixed hadron gas of the π, the ρ and the K at finite temperature (100 MeV $< T <$ 200) (a) the pressure of hadronic matter is plotted as a function of temperature (b) the energy density of hadronic matter is plotted as a function of temperature.

From the above figure 3 (a) and (b), both pressure and energy density increase with an increase in the temperature. The results are in good agreement with those obtained in [4, 12]. The fitted line in both figure 5 and figure 6 represents the power law fit. The focus is on the hadronic scale temperature of (100 MeV $< T <$ 200 MeV) and the zero net baryon number density which is expected to be realized in the central high energy nuclear collisions [4]. The pressure and energy density grow with temperature and start to saturate just after $T = 150$ MeV which might indicate that there is a change in phase transition of the hadron gas. In figure 3 (a) and (b) at low temperatures between $T = 90$ MeV to $T = 150$ MeV the power law T^2 of hadron gas behaves differently than the power law T^4 at high temperature between $T = 155$ MeV to $T = 170$ MeV. The power law of the solid line fit is T^4 which start to behave in the same way as that of hadron matter at high temperatures [18].

5. Conclusion

The results shows that the EoS from the UrQMD model also behaves like results reported by other studies where different model and statistical method was used. The results behaves as expected where we find that the hadronic matter exisit between temperature range 100 MeV $< T <$ 170 MeV. Figure 3 (a) and (b) show that the pressure and the energy density increases exponentially with temperature. The temperature values indicates the phase transion to a new state of matter the QGP at around T = 170 MeV.

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