Third order dissipative fluid dynamics and the Bjorken scaling solution

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Abstract. Third order hydrodynamics equations have been developed using thermodynamics approach. Present calculations are based on entropy principle and the differential equations have been developed in Eckart frame and Bjorken (1+1)D scenario. Energy density, and pressure isotropization etc. as function of proper time have been calculated for massless relativistic fluids. The present calculations have been compared to earlier calculations by A. El et al. and A. Jaiswal et al. An initial QGP formation time of $\tau_0 = 0.4$ fm/c and temperature of $T_0 = 500$ MeV have been used for calculations.

1. Introduction

Heavy ions colliding at relativistic speed form quark gluon plasma (QGP) and give us a scenario similar to early Universe scenario [1]. QGP which is a relativistic fluid with observables such as particle production, elliptic flow are being studied experimentally at RHIC-BNL and LHC-CERN [2–4]. Theoretical transport models on the other hand help us to study the time evolution of particle distributions from the point of collision to freeze-out times when all interaction and production of particles stop. Transport theories include particle interaction and resulting processes such as dissipations, collisions and radiations and are successful in simulating heavy ion collisions and in explaining numerous experimental findings. Earliest works on relativistic fluid dynamics with the first order theories are due to Eckart et al [9]. and to Landau and Lifshitz [10]and Fourier-Navier-Stokes equations were consequently developed. The solutions to first order equations have led to non-causal effects and would propagate viscous and thermal signals with speed greater than that of light. To meet the causality conditions, second order theories were developed by Muller and Israel and Stewart. This is also known as second order dissipative theories or Muller-Israel-Stewart theories (M-IS) [11–13] and [8]. Recent works to include higher order corrections have been done by A. Muronga [14–17], A. El et. al. [18, 19], G. S. Denicol et al. [20], A. Jaiswal et al. [21, 22] etc. using either thermodynamics approach or kinetic Boltzmann equation (BE) to solve the dissipative equations. The results from these various approaches are complimentary to each other [23]. In the current work we have extended the work done by A. Muronga et al. to third order equations for the dissipative fluids [24]. The calculations are shown briefly in section 2. We have compared our calculations and results with earlier calculations by A. El et al. and A. Jaiswal et al. [25–27] who also extended their calculations from second order to third order. The results and discussions are reported in section. 3 of the current manuscript, followed by conclusions.

2. Formalism

The basic formulation of relativistic hydrodynamics can be found in the references mentioned in introduction. Following the prescription by A. Muronga in Ref. [24], we have considered a simple fluid with massless particles and no external electromagnetic fields. The equations for the conservation of net charge $N^{\mu}(x)$ (particle $4-$current), and energy-momentum $T^{\mu\nu}(x)$ (energy $-$ momentum tensor) are then written as

$$\partial_{\mu}N^{\mu}=0, \text{ and } \partial_{\mu}T^{\mu\nu}=0. \tag{1}$$

Also, the second law of thermodynamics dictates for entropy 4-current, S^{μ} is given by,

$$\partial_{\mu}S^{\mu}\geq 0 \tag{2}$$

The genralized form of net charge 4-current might be written in the form,

$$N^{\mu}=nu^{\mu}+v^{\mu} \tag{3}$$

where $n=\sqrt{N^{\mu}N_{\mu}}$ is the net charge density in fluid rest frame and we have considered Eckart's frame where particle flux v^{μ}=0. Then we can calculate $u^{\mu}=\dfrac{N^{\mu}}{\sqrt{N^{\mu}N_{\mu}}}$ as the fluid 4-velocity. In Bjorken (1+1)D expansion it can be shown that $u^{\mu}u_{\mu}=1$. The energy momentum tensor can be shown to be,

$$T^{\mu\nu}=\varepsilon u^{\mu}u^{\nu}-(P+\Pi)\Delta^{\mu\nu}+2q^{(\mu}u^{\nu)}+\pi^{\langle\mu\nu\rangle}\,, \tag{4}$$

where $\varepsilon=u_{\mu}u_{\nu}T^{\mu\nu}$ is the energy density, P is the pressure in fluid rest frame, Π is the bulk viscous pressure, q^{μ} is the heat 4-current and $\pi^{\langle\mu\nu\rangle}$ is shear stress tensor.

In present calculations for relativistic fluid dynamics, we consider a system of gluons and massless quarks that departs slightly from the local thermal distribution. The distribution for particles in that system can then be written as

$$f(x,p)=f^{eq}(x,p)[1+\Delta^{eq}\phi(x,p)]\,, \tag{5}$$

where

$$f^{eq}(x,p)=A_0\frac{1}{e^{\beta_{\nu}p^{\nu}-\alpha}-a}\,, \tag{6}$$

and $\Delta^{eq}=1+aA_0^{-1}f^{eq}(x,p)$ and $\phi(x,p)$ is the deviation/departure function to be discussed shortly. The factor $A_0=g/(2\pi)^3$, where g is the degeneracy factor.

The entropy 4-current can be divided into an equilibrium part and an non-equilibrium part as follows,

$$S^{\mu}(x)=S^{\mu}_{eq}(x)+\delta S^{\mu}(x) \tag{7}$$

To calculate the non-equilibrium part δS^{μ}, we use Grad's 14-field theory with $S^{\mu}(x)$ defined as,

$$S^{\mu}(x)=-\int dw\, p^{\mu}\psi(f), \tag{8}$$

where

$$\psi(f)=f(x,p)\ln[A_0^{-1}f(x,p)]-a^{-1}A_0\,\Delta(x,p)\ln\Delta(x,p) \tag{9}$$

The function $\psi(f)$ has been expanded around equilibrium distribution function $f^{eq}(x,p)$. $\psi(f^{eq})$ in the expansion gives the equilibrium part of the entropy while the rest of the expansion

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is used to derive its' non-equilibrium part. A small linear departure function $\phi(x,p)$ is used in $f(x,p)$, with quadratic dependence on 4-momentum as (for detail calculation see Ref. [28]),

$$\phi(x,p) \approx \epsilon(x) - \epsilon_\mu(x)p^\mu + \epsilon_{\mu\nu}(x)p^\mu p^\nu \tag{10}$$

The moments ϵ, ϵ_μ and $\epsilon_{\mu\nu}$ are assumed small.

After integration, the entropy 4-current can be written up to third order or cubic in dissipative fluxes as,

$$\begin{aligned} S^\mu &= S_1^0 u^\mu + S_1^1 \Pi u^\mu + S_2^1 q^\mu + \left(S_1^2\Pi^2 - S_2^2 q^\alpha q_\alpha - S_3^2 \pi^{2\langle\alpha\alpha\rangle}\right)u^\mu + S_4^2 \Pi q^\mu \\ &+ S_5^2 \pi^{\langle\mu\alpha\rangle} q_\alpha + \left(S_1^3\Pi^3 - S_2^3 \Pi q_\alpha q^\alpha + S_3^3 \Pi \pi^{2\langle\alpha\alpha\rangle} + S_4^3 q_\alpha q_\beta \pi^{\langle\alpha\beta\rangle} + S_5^3 \pi^{3\langle\alpha\alpha\rangle}\right)u^\mu \\ &+ \left(S_6^3\Pi^2 - S_7^3 q_\alpha q^\alpha + S_8^3 \pi^{2\langle\alpha\alpha\rangle}\right)q^\mu + S_9^3 \Pi \pi^{\langle\mu\alpha\rangle} q_\alpha + S_{10}^3 \pi^{2\langle\mu\alpha\rangle} q_\alpha \end{aligned} \tag{11}$$

where the coefficients S_n^m are calculated as functions functions of (ε and n) and will be shown elsewhere. The coeffcients will be shown elsewhere. The superscript in the coefficients denotes the order and the subscript labels the coefficient number in that order. For thermodynamic processes, the entropy principle suggests, $\partial_\mu S^\mu \geq 0$. The dissipative fluxes can be obtained either from the equations of the balance of the fluxes or from entropy principle.

The calculations are done in Bjorken (1+1)D scenario where $u^\mu = (\frac{t}{\tau}, 0, 0, \frac{z}{\tau})$ and a baryon chemical potential free, $\mu_c = 0$ has been considered. Eckart frame has been assumed and also in Bjorken scaling solution heat flow can be shown to be $q^\mu = 0$ [18]. In the case of massless particles, bulk viscosity can also be neglected while bulk pressure equation does not contribute.

Thus from the entropy principle, the transport equation for shear viscous pressure could be reduced to (see Ref. [28] for detailed calculation)

$$\begin{aligned} \pi &= \frac{4}{3}\frac{\eta}{\tau}, \quad (1^{\rm st}\ \text{order}) \\ \dot{\pi} &= -\frac{\pi}{\tau_\pi} - \frac{1}{2}\frac{\pi}{\tau} + \frac{3}{10}\frac{\varepsilon}{\tau} - \frac{3}{2}\frac{\pi^2}{\varepsilon\tau} + \frac{5}{8}\frac{\pi}{\varepsilon}\dot{\varepsilon} + \frac{27}{8}\frac{\pi^2}{\varepsilon^2}\dot{\varepsilon} - \frac{6}{5}\frac{\pi}{\varepsilon}\dot{\pi}, \quad (\text{upto } 3^{\rm rd}\ \text{order}) \end{aligned} \tag{12}$$

For the (1+1) dimensional Bjorken flow in (3+1) dimensions the energy equation is given by,

$$\dot{\varepsilon} = -\frac{4}{3}\frac{\varepsilon}{\tau} + \frac{\pi}{\tau}. \tag{13}$$

where $\tau_\pi = 2\eta S_3^2$ is relaxation time for the shear pressure. The coefficient, S_3^2 is taken to be $\sim 9/4\varepsilon$ in the ultra-relativistic limits. We have used equation of state (EoS) due to assumed ultra-relativistic scenario to be, $\varepsilon = 3P$. Next we move to results and discussion section.

3. Results and Discussion

In Fig. 1 we have shown pressure isotropy ratio, P_L/P_T and compared the results from present model by A. Muronga with earlier third order models by A. El et al. and A. Jaiswal et al which is based on kinetic theory approach to Boltzmann transport equation in relaxation time approximation. The shear equations in the current model have been derived from full entropy 4-current expression without neglecting any shear terms (*viz.* non-linear terms neglected in Israel-Stewart theory have been included). Two different values of $\eta/s = 0.1$ and 0.5 have been taken to illustrate the differences between these three models. For $\eta/s = 0.1$, the present

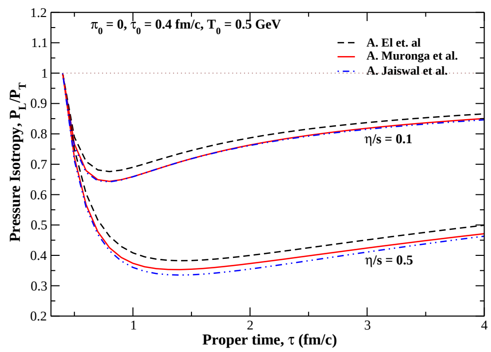

Figure 1: Comparison of models on pressure isotropy ratio for two different values of η/s.

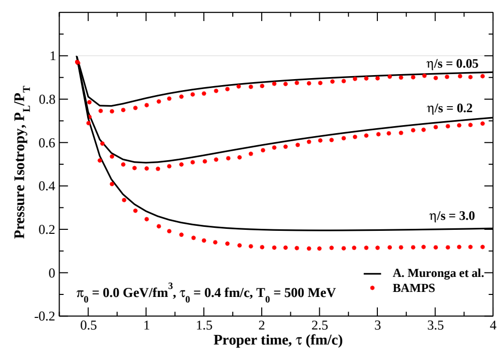

Figure 2: η/s dependence of pressure isotropy ratio from current model/work

calculation is closer to results from A. El et al. While for modestly high value of $\eta/s = 0.5$, the results from present calculation appear to be closer to A. Jaiswal's model. However it should be noted that the values of the coefficients in shear equations are different in all three models. The solutions to the equations depends on the values of the coeffcients although the differences are not considerable. The differences might arise from various initial approximations and assumptions. They tend to bring in uncertainties in the final solutions and outputs. It can also be noted that the models of A. Muronga et al and A. El differ in the number of terms considered for the shear differential equation. Various values of η/s parameter could be taken to illustrate the differences in the models.

In Fig. 2, we have shown the ratio η/s calculated from present model (A. Muronga) for three different values of viscosity to entropy ratio of 0.05, 0.2 and 3.0. The current work has been compared to BAMPS transport calculation. We have also chosen initial shear pressure, $\pi_0 = 0$. This particular initial condition for shear gives ideal scenario for the system initially with P_L/P_T being unity at starting point. Thereafter system develops shear pressure immediately and goes out of equilibrium. However the particles within the system interact and bring down the shear effects with time and the system tends to return to equilibrium once again. We find that for lower values of the parameter the isotropy ratio tends to return to unity which suggests that the system may return to equilibration if given enough time. While for a large value of $\eta/s = 3.0$, thee ratio is almost flat after 2 fm/c and system may not return to equilibrium within the lifetime of QGP. The present third order model also underestimate the transport results due to BAMPS by a small magnitude but the shape of the curves are similar.

In Fig. 3, we have shown energy density of quark gluon plasma as a function of proper time. A modest value for viscosity to entropy ratio, $\eta/s = 6/4\pi$ has been used. We have also used two different initial values for $\pi_0 = 4\eta/3\tau_0$ and 0.0 GeV/fm^3. The ideal fluid equation gives energy density which fall as $\sim \dfrac{1}{\tau^{4/3}}$ [29]. The first order dissipative equation gives a rise in the energy density initially and then falls the slowest. The higher orders bring down the rise in energy density with third order being closest to ideal scenario. However the time evolution of energy density shows a strong dependence on values chosen for initial shear pressure, π_0.

4. Conclusion

Third order shear equations have been developed after extending earlier calculations by A. Muronga. The energy density as function of proper time shows effect of different ordered theo-

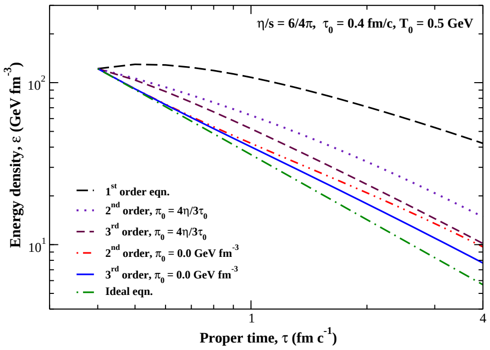

Figure 3: Comparison of orders on energy density

ries. Ideal equations brings down the evolving energy density quickest while first order theory is slowest. Both second and third order theories brings down the magnitude quit close to ideal scenario but choice of initial conditions have considerable effect on the observable. A systematic study of dependence of solutions on initial conditions will be conducted. It is also shown that dissipative fluxes tend to put the system out of local thermal equilibrium but system tends to go back to being ideal state. The value of η/s play a vital role in this. The present calculations have also been compared to other third order models and transport theory of BAMPS. The difference in the outputs with third order models are due to different values of coefficients in the equations and also due to different number of terms considered within the differential equations. This particular aspect is interesting and will be studied in detail.

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