Computation of the effective potential in the gauge-Higgs unification model with an $SU(3)$ representation

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Abstract. Gauge-Higgs unification models give interesting solutions to the hierarchy problem in particle physics. The common study of this type of model is done by using a decomposition of 5-dimensional particles in 4-dimensional Kaluza-Klein modes, which is a handy way to compute the infinite sums appearing in the model. In order to take into account the running of coupling constants in these models, we propose in this proceedings a different decomposition using winding modes around the fifth dimension, which is compactified. This decomposition not only permits us to take running into account, but also gives a faster converging series in all the quantities when summing over these modes.

1. Introduction

Since the discovery of a Higgs boson at the Large Hadron Collider (LHC) in 2012, a lot of questions have emerged concerning its mass and couplings, as they are close to the electroweak (EW) symmetry breaking scale. One such issue relates to the Planck scale being 10^{24} times higher than the EW scale, and that this scale enters the quantum loop corrections to the Higgs boson mass, giving rise to the hierarchy problem. This problem can be solved for gauge theories in more than four-dimensions [1], which can also give an interesting unification of gauge and Yukawa couplings with the running in the renormalisation group [2], where we can see that all the coupling constants run towards a common value at a large energy scale.

These theories include the Higgs boson as a component of a multidimensional gauge boson, rather than via an *ad hoc* addition to the model. In a previous work by some of the authors, an interesting toy model was developed, that of a flat 5-dimensional space-time compactified as a S^1/Z_2 [2]. In this model there exists different methods for studying the 5-dimensional fields, where the most common is to decompose the fields in an infinite tower of 4-dimensional fields (as a Fourier decomposition) called Kaluza-Klein modes (KK modes). This decomposition is convenient, as it allows us to compute (in simple cases) the infinite sums, and is the most common approach used in the literature [3, 4].

In order to introduce the running of the coupling constants in this kind of decomposition, we need to add contributions it at the right energy scale for each KK mode, and this complicates the calculation. We will present a different decomposition, in terms of winding modes, allowing a simpler physical interpretation. Whilst this method may appear to be harder to work with, as each term can be taken at a specific energy scale, it does enable a full running calculation to be done. This winding mode approach is further explained in references [5, 6]. Moreover, we will see that this decomposition brings advantages in terms of not just renormalisation, but also for the convergence of the infinite sums. As such, in section 2, we will see how the gauge-Higgs model can be described in an $SU(N)$ representation, deriving the effective Higgs potential in terms of winding modes in section 3. Finally, in section 4, we will study the effective potential and what can be done to improve the $SU(3)$ model studied here.

2. Gauge-Higgs unification with an S^1/Z_2 orbifold

According to reference [3], we can develop a gauge-Higgs unification model on a five-dimensional orbifold $M^4 \times (S^1/Z_2)$, where M^4 is the 4-dimensional Minkowski-space and S^1/Z_2 is obtained by identifying two points on the compactified fifth dimension S^1 by parity for $x = 0$ and $x = \pi R$. Our model is defined by the boundary conditions and by the parity operators defined as:

$$\begin{aligned} U : A_M(x, y + 2\pi R) &= U A_M(x, y) U^\dagger \ , &(1)\\ P_0 : A_\mu(x, -y) &= P_0 A_\mu(x, y) P_0^\dagger \ , &(2)\\ P_0 : A_y(x, -y) &= -P_0 A_y(x, y) P_0^\dagger \ , &(3)\\ P_1 : A_\mu(x, \pi R - y) &= P_1 A_\mu(x, \pi R + y) P_1^\dagger \ , &(4)\\ P_1 : A_y(x, \pi R - y) &= P_1 A_y(x, \pi R + y) P_1^\dagger \ , &(5) \end{aligned}$$

where A_M is a gauge field in 5-dimensions, with the convention that we use Greek letters for the 4-dimensions of M^4 and Latin letters for 5-dimensions (or just 5 for the fifth dimension). Normally we have $U = e^{i\alpha} P_1 P_0$, but as it does not affect the results, and for simplicity, we will take $U = P_1 P_0$.

From these operators it is possible to define the boundary conditions for the other fields in our model, where for a scalar field ϕ we have

$$\begin{aligned} \phi(x, y + 2\pi R) &= e^{i\pi\beta_\phi} T_\phi[U] \phi(x, y) \ , &(6)\\ \phi(x, -y) &= \pm T_\phi[P_0] \phi(x, y) \ , &(7)\\ \phi(x, \pi R - y) &= \pm e^{i\pi\beta_\phi} T_\phi[P_1] \phi(x, \pi R + y) \ , &(8) \end{aligned}$$

where $T[U]$ represents an appropriate representation matrix. For instance, if ϕ belongs to the fundamental or adjoint representation of the group, then $T_\phi[U]\phi$ is $U\phi$ or $U\phi U^\dagger$, respectively. Note that $e^{i\pi\beta_\phi}$ must be equal to either +1 or -1.

For Dirac fields ψ we have

$$\begin{aligned} (x, y + 2\pi R) &= e^{i\pi\beta_\psi} T_\psi[U] \psi(x, y) \ , &(9)\\ (x, -y) &= \pm T_\psi[P_0] \gamma^5 \psi(x, y) \ , &(10)\\ (x, \pi R - y) &= \pm e^{i\pi\beta_\psi} T_\psi[P_1] \gamma^5 \psi(x, \pi R + y) \ , &(11) \end{aligned}$$

where as before, we must have $e^{i\pi\beta_\psi}$ to be equal to +1 or -1.

The following Lagrangian is then used in order to compute the effective potential:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} \ ; \qquad (12)$$

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$$\mathcal{L}_{gauge} = -\frac{1}{2}\text{Tr}(F_{MN}F^{MN}) - \frac{1}{\alpha}\text{Tr}(F[A]^2) - \text{Tr}\left(\overline{\eta}\frac{\delta F[A]}{\delta A_M}D^M\eta\right) \ , \quad (13)$$

$$\mathcal{L}_{matter} = \overline{\psi}i\gamma_M D^M\psi + |D_M\phi|^2 \ . \quad (14)$$

We then split the gauge field A_M into its classical part A_M^0, and its quantum part A_M^q, from which we have the gauge-fixing condition $F[A^0] = 0$, such that

$$F[A] = D_M(A^0)A^{qM} = \partial_M A^{qM} + ig[A_M^0, A^{qM}] = 0 \ . \quad (15)$$

Note that the notation $D_M(A^0)$ is often denoted D_M^0 for short. This means that $\mathcal{L}_{gauge}$ can be rewritten as:

$$\mathcal{L}_{gauge} = -\text{Tr}(A_M^q M_{MN}^g A^{Nq}) - \text{Tr}(\overline{\eta}M^{gh}\eta) \ , \quad (16)$$

$$\text{where} M_{MN}^g = -\eta_{MN}D_L^0 D^{0L} - 4igF_{MN}^0 \ , \quad (17)$$

$$M^{gh} = D_L^0 D^{0L} \ . \quad (18)$$

Integrating out the quantum fields A_M^q, η, $\overline{\eta}$, and ϕ, we obtain the one-loop effective potential for A_M^0:

$$V_{eff}[A^0] = V_{eff}[A^0]^{g+gh} + V_{eff}[A^0]^{fermion} + V_{eff}[A^0]^{scalar} \ ; \quad (19)$$

$$V_{eff}[A^0]^{g+gh} = -(D-2)\frac{i}{2}\text{Tr}(\ln(D_L^0 D^{0L})) \ , \quad (20)$$

$$V_{eff}[A^0]^{fermion} = h(D)\frac{i}{2}\text{Tr}(\ln(D_L^0 D^{0L})) \ , \ h(D) = 2^{D/2} \ , \quad (21)$$

$$V_{eff}[A^0]^{scalar} = -2\frac{i}{2}\text{Tr}(\ln(D_L^0 D^{0L})) \ , \quad (22)$$

where we have supposed that $F_{MN} = 0$ and ϕ-fields are massless. From the next section onwards we shall focus on a particular group to compute the effective potential.

3. Functional method in $SU(3)_w$

As described in reference [7], it is possible to take $P_1 = P_2 = \text{diag}(1, -1, -1)$ to break the group $G = SU(3)_w \times SU(3)_c$ to $G_{SM} = SU(2)_w \times U(1) \times SU(3)_c$, which is exactly the Standard Model group. In this configuration we only have a doublet for A_y, belonging to G/G_{SM}, which has a zero-mode. This doublet can be identified as our Higgs, such that $H = (A_y^{1(0)} + iA_y^{2(0)}, A_y^{4(0)} + iA_y^{5(0)})^t$ where the index i in the notation A_y^i refers to the $SU(3)$ generator index. The vacuum expectation value (VEV) is a finite calculable quantity, which is determined by the minimisation of the one-loop induced effective potential as the function of a constant background field, $\langle A_y \rangle \equiv B_y$. In this case, keeping only the non-vanishing zero-modes and using the global $SU(2) \times U(1)$ symmetry, we can set the VEV in the form of $B_y^a = (B_y^1, 0, 0, 0, 0, 0, 0, 0)$, which leads to the effective potential formula:

$$V_{eff} = V_{eff}^{g+gh} + V_{eff}^{f} \ ; \quad (23)$$

$$V_{eff}^{g+gh} = \frac{3}{2}\frac{1}{2\pi R}\int\frac{d^4p}{(2\pi)^4}\sum_{n=-\infty}^{\infty}\left[\ln\left(-p^2 + \frac{n^2}{R^2}\right) + \ln\left(-p^2 + \left(\frac{(n-\alpha)}{R}\right)^2\right) + 2\ln\left(-p^2 + \left(\frac{(n-\alpha/2)}{R}\right)^2\right)\right] \ , \quad (24)$$

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$$V_{eff}^{f} = -N_f \frac{1}{2\pi R} \int \frac{d^4p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \left[\ln\left(-p^2 + \frac{n^2}{R^2}\right) + 2\ln\left(-p^2 + \left(\frac{(n-\alpha/2)}{R}\right)^2\right)\right] . \tag{25}$$

The variable α is proportional to the VEV of B_y where $B_y = \alpha/gR$ [7]. N_f is the number of fermions we consider in this model, and the sum over n is over the full KK tower.

As explained in reference [6], it is possible to replace the KK modes in the previous expression by the winding modes we want to use. To do this we can identify in the propagator expression for the KK decomposition:

$$\sum_{n=-\infty}^{+\infty} \ln\left[-p^2 + \left(\frac{n-m_0}{R}\right)^2\right] = -\int dp^2 \sum_{n=-\infty}^{+\infty} \frac{1}{-p^2 + \left(\frac{n-m_0}{R}\right)^2} = \int dp^2 \tilde{G}_{KK}(p, m_0) , \tag{26}$$

where m_0 represents the mass of the zero-mode and $\tilde{G}_{KK}(p, m_0)$ represents the KK propagator of a scalar particle. From this we can replace the propagator in KK modes by the propagator in S^1/Z_2 in terms of winding modes, such that:

$$\int dp^2 \tilde{G}_{KK}(p, m_0) = \int dp^2 \int_0^{\pi R} dy \sum_{n=0}^{+\infty} [\tilde{G}_{Winding}(p, 2n\pi R, m_0) \pm \tilde{G}_{Winding}(p, 2y + 2n\pi R, m_0)] , \tag{27}$$

where $\tilde{G}_{Winding}(p, |y-y'|, m_0)$ is the winding mode propagator and can be written as:

$$\tilde{G}_{Winding}(p, |y-y'|, m_0) = \frac{e^{i\chi|y-y'|}}{2\chi} , \tag{28}$$

with $\chi = \sqrt{p^2 - m_0^2}$.

We can now do a different type of regularisation here by just taking out the winding mode $n = 0$, which removes the whole divergent part of the effective potential. From this, we can replace the propagators by their simplified integrals:

$$\int dp^2 \int_0^{\pi R} dy \sum_{n=0}^{+\infty} [\tilde{G}_{Winding}(p, 2n\pi R, m_0) \pm \tilde{G}_{Winding}(p, 2y + 2n\pi R, m_0)] = \int dp^2 \sum_{n=0}^{+\infty} \left[\frac{\pi R e^{i\chi 2\pi R n}}{2\chi} \pm \frac{e^{i\chi 2\pi R(n+1)} - e^{i\chi 2\pi n R}}{4i\chi^2}\right] . \tag{29}$$

We now perform a Wick rotation ($i\chi \to -\chi_E$) to perform a Euclidian integral. We also have that $dp^2 = d\chi^2 = 2\chi d\chi$ = -2$\chi_E d\chi_E$, which means that

$$\int dp^2 \sum_{n=1}^{+\infty} \left[\frac{\pi R e^{i\chi 2\pi R n}}{2\chi} \pm \frac{e^{i\chi 2\pi R(n+1)} - e^{i\chi 2\pi n R}}{4i\chi^2}\right] = \int d\chi_E \sum_{n=1}^{+\infty} e^{-\chi_E 2\pi R n} \left[\pi R \mp \frac{e^{-\chi_E 2\pi R} - 1}{2\chi_E}\right] = -\sum_{n=1}^{+\infty} \left(\frac{e^{-\chi_E 2\pi R n}}{2n} \pm \pi R (E_i(-2\pi R(n+1)\chi_E) \mp E_i(-2\pi R n \chi_E))\right) , \tag{30}$$

where $\chi_E = \sqrt{p_E^2 + m_0^2}$ and $E_i(x)$ is the exponential integral function defined as

$$E_i(x) = -\int_{-x}^{+\infty} \frac{e^{-t}}{t} .$$

From this, we finally have that:

$$-3\frac{i}{2}\int \frac{dp^4}{(2\pi)^4} \frac{1}{2\pi R}\left\{\sum_{n=-\infty}^{+\infty} \ln\left[-p^2+\left(\frac{n-m_0}{R}\right)^2\right]\right\} =$$
$$-\frac{3}{4} \int dp_E \frac{p_E^3\Omega(4)}{(2\pi R)^4}\sum_{n=1}^{+\infty}\left(\frac{e^{-\chi_E 2\pi Rn}}{2\pi Rn} \pm (E_i(-2\pi R(n+1)\chi_E) \mp E_i(-2\pi Rn\chi_E))\right) , \tag{31}$$

where $\Omega(4)$ is the volume of the 4-dimensional sphere, which is equal to $\pi^2/2$. We define the function $f(m,n)$ such that:

$$f(m_0,n) = -\frac{3}{128}\int dp_E \frac{p_E^3}{\pi^2 R^4}\left(\frac{e^{-\chi_E 2\pi Rn}}{2\pi Rn} \pm (E_i(-2\pi R(n+1)\chi_E) \mp E_i(-2\pi Rn\chi_E))\right) . \tag{32}$$

Now we can rewrite the effective potential with winding modes decomposition, using the f function which is defined for all m_0 and $n > 0$ as:

$$V_{eff}(\alpha) = \sum_{n=1}^{\infty}\left[f(0,n) + f\left(\frac{\alpha}{R},n\right) + 2f\left(\frac{\alpha}{2R},n\right) - \frac{2}{3}N_f\left(f(0,n)+2f\left(\frac{\alpha}{2R},n\right)\right)\right] . \tag{33}$$

The advantage of this expression is that, in each mode n, the running coupling constant can be taken at the energy scale of the mode, so that we can easily take into account the running developed in reference [2] for this $SU(3)$ model of gauge-Higgs unification. Moreover, each term in n can be calculated numerically and the series converges much faster than for the KK mode approach. For example, the evaluation for $n = 1$ and $n = 2$ shows that there is a factor of 100 between them, due to the exponential dependence on n in each term. This can be compared to the KK sum, which only converges as $1/n^5$. As such, it is possible to effectively study the global evolution of the effective potential with α only with the first term $n = 1$, or with the first few terms.

In figure 1 we find that for $N_f = 1$ the effective potential is symmetric in α, and the only minimum for the first term of $V_{eff}(\alpha)$ is for $\alpha = 0$. This means that we don't have any spontaneous symmetry breaking in this model. We can argue that the shape of the potential is different from the one using KK modes in reference [7], however, this is due to the different approximations used for the KK modes, such as the regularisation method, which means

$$\int \frac{d^4p_E}{(2\pi)^4}\sum_{n=1}^{\infty}\log\left(p_E^2+\frac{(n-\alpha)^2}{R^2}\right) \to \sum_{n=1}^{\infty}\frac{1}{n^5}\cos(2\pi n\alpha) . \tag{34}$$

Note that the periodicity of the effective potential comes from this approximation.

4. Conclusion

In this paper we have described a new method for computing the effective Higgs potential in a gauge-Higgs unification model. This method allows us to take into account the running of the coupling constants inside each term of the effective potential. In our toy model with $SU(3)$ the running doesn't have a great deal of impact, because the coupling constants decrease when n goes up, which means that the running makes the sum converge even faster, and so the shape of the effective potential with $n = 1$ does not change much compared to the full effective potential. As such, it seems that this simple model gives a potential that does not provide spontaneous symmetry breaking. On the other hand, this method does permit a simple

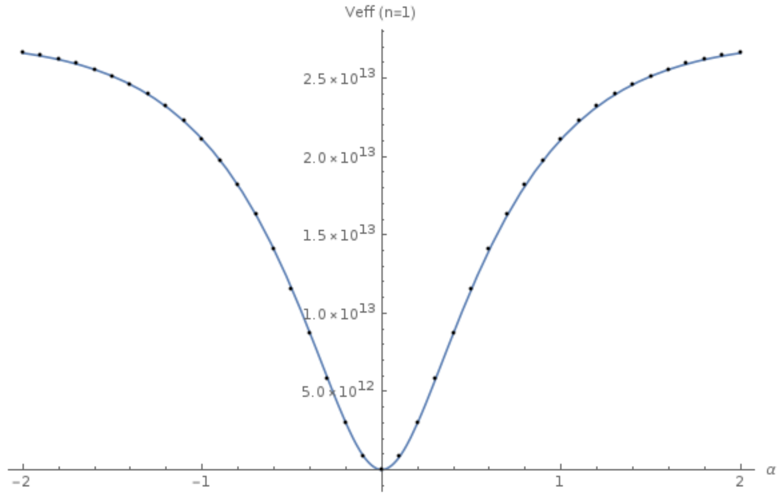

Figure 1. First term ($n = 1$) of the effective potential with $N_f = 1$ for different values of α.

elimination of divergences by removing the $n = 0$ mode, and the convergence of the sum is faster than with KK modes. Moreover, we didn't use any approximation on the VEV compared to the compactification scale to do the calculation, which means that our effective potential remains accurate for higher values of α, that is, it will remain a valid effective description.

To extend beyond this work we have begun using this method with an $SU(5)$ model, along with attempts with other methods: such as diagrammatic computation of the effective potential with winding modes, functional computation with KK modes, differential computation of the effective potential. Note that each of these other methods have major drawbacks making it difficult to include the running of coupling constants. A discussion of these methods shall be the focus of a future work [8].

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