



UNISA



**International symposium on New Developments in Methods and Applications  
of Few-body Physics: in Memory of Professor SA Sofianos**

# **SOFIANOS2017**

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## **Book of abstracts**

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**Oral Sessions / 0**

**INVITED SPEAKER: Modelling the progenitors and environments of stellar explosions**

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Just Testing

**Oral Sessions / 2**

**Artificial Neural Networks with Infinite number of nodes.**

**Author:** Prof. LAGARIS, Isaac <sup>1</sup>

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A new class of Artificial Neural Networks is described incorporating a node density function and functional weights.

This network containing an infinite number of nodes, excels in generalizing and possesses a superior extrapolation capability.

## Invited Talks / 3

**Scaling behavior of scattering observables for three-body systems near the unitary limit**Prof. TOMIO, LAURO <sup>1</sup><sup>1</sup> *INSTITUTO TECNOLÓGICO DE AERONÁUTICA***Corresponding Author:** tomio@ift.unesp.br

Low-energy scaling properties of three-body systems are investigated, by considering the elastic  $s$ - $s$ -wave collision of a particle in a bound-state formed by the remaining two-body system. In the first part of my presentation I will concentrate in the case of the halo nucleus  $^{20}\text{C}$ , where we have examined the neutron- $^{19}\text{C}$  scattering properties near the critical condition for the occurrence of an excited bound state in  $^{20}\text{C}$  (within a neutron-neutron- $^{18}\text{C}$  configuration), by considering zero-range and finite-range interactions [1] for the two-body subsystems. The results for the  $s$ - $s$ -wave scattering amplitude present universal scaling features, with the variation of the  $^{19}\text{C}$  binding energy for fixed  $^{20}\text{C}$  binding and neutron-neutron singlet virtual state energies. The scaling of the effective-range parameters and the pole position of the scattering observable  $k \cot(\delta_0^R)$  (where  $k$  is the momentum corresponding to the colliding energy and  $\delta_0^R$  is the real part of the  $s$ - $s$ -wave phase shift) are in general consistent with the scaling obtained with a zero-range potential. Next, by considering the actual possibilities for verification of low-energy scaling properties in cold-atom laboratories, I am going to consider strongly-mass-imbalanced three-body atomic systems, with the collision of a heavy particle in the light-heavy weakly-bound system. Our preliminary results [2] for scattering observables, obtained with zero-range interactions for the two-body bound system and no interaction between the two-heavy particles, are evidencing the universal scaling features.

References:

[1] M.A. Shalchi, M.T. Yamashita, M.R. Hadizadeh, T. Frederico, L. Tomio, Neutron- $^{19}\text{C}$  scattering: Emergence of universal properties in a finite-range potential, *Phys. Lett. B* **764** (2017) 196-202.

[2] M.A. Shalchi, M.T. Yamashita, M.R. Hadizadeh, L. Tomio, T. Frederico, Probing the Efimov scaling for atom-molecule scattering, in preparation.

## Oral Sessions / 6

**Efimov states in asymmetric trimers**Dr. KOLGANOVA, Elena <sup>1</sup><sup>1</sup> *BLTP JINR***Corresponding Author:** kea@theor.jinr.ru

The van der Waals three-body systems at ultralow energies are studied using Faddeev equations in configuration space. The spectra of LiHe2 are calculated. The results obtained indicate on the Efimov character of the excited states.

Oral Sessions / 7

## An equivalence of the complex rotation resonances and scattering matrix resonances

Prof. MOTOVILOV, Alexander <sup>1</sup>

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We consider a momentum-space multi-channel Hamiltonian that underwent the complex rotation, a kind of inhomogeneous complex scaling. Isolated non-real eigenvalues of this Hamiltonian are called the complex rotation resonances. For a class of sufficiently rapidly decreasing and analytic interactions, we prove that the complex rotation resonances do correspond to the scattering matrix resonances, that is, to the poles of the scattering matrix analytically continued to the respective unphysical sheet. Our proof employs the explicit representations that express the multi-channel T- and S-matrices on unphysical energy sheets through those same matrices taken only in the physical sheet.

Oral Sessions / 8

## Elastic breakup of ${}^6\text{Li}$ on different targets

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We study the  ${}^6\text{Li}$  breakup on different target masses in order to investigate the dependence of continuum-continuum couplings and Coulomb-nuclear interference on the target mass. We show that excluding the continuum-continuum couplings, the integrated total and nuclear breakup cross sections decrease linearly as function of  $A_T^{1/3}$ , while the integrated Coulomb breakup decrease linearly as function of the target charge. The Coulomb-nuclear interference scales linearly as function of the target charge when all the different couplings are included in the potential matrix element.

## Oral Sessions / 9

**Jost function method approach for study of unstable nuclei**Prof. MASUI, Hiroshi <sup>1</sup><sup>1</sup> *Kitami Institute of Technology***Corresponding Author:** hgmasui@mail.kitami-it.ac.jp

We have developed the formalism of the Jost function method (JFM) to study unstable nuclei. The component of the unbound states becomes important in the field of quantum systems where open channels are included in the system, e.g. study of unstable nuclei. From the aspect of the few-body treatment, the position of the S-matrix pole is a crucial key to characterize the system. The pole position can be obtained very precisely by using the Jost function method (JFM). A practical recipe for the numerical treatment of JFM was given by S. A. Sofianos and S. A. Rakityanskiy. Inspired by this practical approach, we have developed the formalism of the Jost function method (JFM) to study the partial decay widths in coupled-channel systems, virtual (anti-bound) states, non-local kernels in the Hamiltonian and application to the Lagrange-mesh formalism. In this talk, we show the applications of JFM for studying the unstable nuclei.

First, we have shown the partial decay widths in a coupled-channel system can be determined by JFM under the assumption that the sum of the partial widths becomes the total width. We demonstrate how accurately we can determine the partial decay widths even for broad resonant cases and show that two different schemes to define the widths give exactly the same result using the JFM approach.

Next important progress on the study of unstable nuclei is the discussion of the virtual state pole within the other physical observables, i.e. scattering phase-shift and scattering length. We compare the two systems,  $^5\text{He}$  and  $^{10}\text{Li}$ , which are considered to be the subsystem of halo nuclei:  $^6\text{He}$  and  $^{11}\text{Li}$ . To understand the structure of  $^{11}\text{Li}$ , the position of the pole in s-wave state of  $^{10}\text{Li}$  is important. Because  $^{11}\text{Li}$  is a p-shell nucleus in the shell model point of view, and the valence neutrons are in the  $0p_{1/2}$ -orbit with respect to the  $^9\text{Li}$  core. Nevertheless, experiments indicate the strong s-wave component for the low-lying state of  $^{10}\text{Li}$  ( $^9\text{Li}+n$ ) system, and no s-wave bound states are observed. Hence, such the s-state can be considered as a virtual (anti-bound) state. For the theoretical approach to study of  $^{11}\text{Li}$ , we need to determine the potential strength of the  $^9\text{Li}+n$  system. The position of the S-matrix pole can be investigated by JFM even for the virtual states, and we discuss how the pole moves on the complex momentum plane by changing the potential strength. The results are connected to other physical observables such as the phase shift and scattering length.

The remaining issues for the JFM approach is to include non-local potentials or kernels into the formalism. We proposed a practical approach for solving an integro-differential equation of JFM.

Using this formalism, the virtual state of  $^{10}\text{Li}$  can be determined including the non-local Pauli projection operators under the orthogonality condition model. Recently, we apply JFM to the Lagrange-mesh approach. The prominent advantages of the Lagrange-mesh approach are follows. The mesh points of this method can be taken very small number compared to the conventional numerical integration for solving the differential equation, e.g. the Runge-Kutta method. The typical number of the mesh points is 20 to 100. Furthermore, the mesh points are defined as the zero-point of the Lagrange polynomials. Hence, the non-local potential which induces the integration in the differential equation can be reduced to a value to each mesh point.

## Oral Sessions / 10

**Gaussian Expansion Method and its application to few-nucleon systems**Prof. HIYAMA, Emiko <sup>1</sup><sup>1</sup> *Nishina Center, RIKEN, Wako, 351-0198, Japan***Corresponding Author:** brian.masara@saip.org.za

One of the most important subjects in physics is to calculate few-body Schroedinger equation accurately. By solving the equation, we can predict various observable before measurement and can obtain new understanding. For this purpose, it is necessary to develop the method to calculate three- and four-body problems precisely and to apply to various fields such as nuclear physic as well as atomic physics.

We proposed 'Gaussian Expansion method using infinitesimally-shifted Gaussian lobe basis function'. This method has been applied to few-nucleon systems, hadronic systems, atomic systems etc.

In this symposium, I will introduce the Gaussian Expansion method and report one of recent hot topics, tetra neutron system. The structure of tetra neutron system was already done by Prof. Sofianos. Here, I will report what is progress after his work.

## Oral Sessions / 11

**A New Holizon of Few-Body Problems Exact Coulomb treatment and the energy-momentum translation of the three-body Faddeev equations**Prof. SHINSHO, Oryu <sup>1</sup><sup>1</sup> *Department of Physics, Faculty of Science and Technology, Tokyo University of Science, Yamazaki, Noda City, Chiba 278-8510 Japan***Corresponding Author:** brian.masara@saip.org.za

Our two recent developments will be presented.

1) We confirm the reliability of the well-known Alt's Coulomb renormalization method (CRM). It is found that the CRM is only available for a very- long-range screened Coulomb potential (SCP), where asymptotic property can be satisfied. However, such an SCP calculation in momentum space is considerably difficult because of the cancelation of significant digits. In contrast to the CRM, we propose a new method by using an on-shell equivalent SCP and the rest term. We introduce the two-potential theory with r-space, which defines fully off-shell Coulomb amplitude. We obtain the Coulomb phase shift with nine~ten digit accuracy. Our method can reproduce any charged particle systems from electron-electron to heavy ion-heavy ion systems such as  $^{208}\text{Pb}$ - $^{208}\text{Pb}$ .

2) An energy-momentum translation in the three-body Faddeev equation, with a proposed extension of the integral domain of the conventional Faddeev equations, is carried out. We found that the quasi-two-body threshold

at the energy  $E = -\epsilon B$  for the reaction process  $(a, b, c) \rightarrow a + (b, c)$  di-

verges, and so does the three-body break up threshold at the three-body energy  $E = 0$  for the process  $a + (b, c) \rightarrow a + b + c$ . Furthermore, an

analytic continuation from the three-body Faddeev equations to the multi-channel quasi-two-body Lippmann-Schwinger equations is performed. The divergence at the quasi-two-body threshold plays an essential role for solving the scattering length in a three-body system. In our new procedure, the three-body binding energy can be redefined.

The author is indebted to the late Professor S. A. Sofianos for his continuous encouragement from the beginning of this work.

## Oral Sessions / 13

**Partial widths of a multi-channel resonance**Prof. RAKISTIANSKI, Sergi <sup>1</sup><sup>1</sup> UP**Corresponding Author:** brian.masara@saip.org.za

A new method for finding the partial decay-widths for multi-channel resonances is suggested. It is based on using the Jost matrices. These matrices can be either calculated (when the multi-channel potential is known) or found by fitting experimental data (when they are available). In both cases the procedure for obtaining the partial widths is the same and is simple. It does not require any kind of integration or differentiation. The partial widths sum up to the total width and the corresponding branching ratios can be found as some algebraic combinations of the elements of the Jost matrices.

## Oral Sessions / 14

**Few-body integrodifferential equation on Lagrange-mesh**Prof. RAMPHO, Gaotsiwe Joel <sup>1</sup><sup>1</sup> University of South Africa**Corresponding Author:** ramphjg@unisa.ac.za

The two-variable integrodifferential equation for few-body systems is solved using the Lagrange-mesh method. The method transforms the equation into a system of algebraic equations that are solved as a non-symmetric matrix eigenvalue problem. Convergence properties of the solution in relation to the problem parameters is investigated. The accuracy of the converged solution is tested by calculating the binding energies and root-mean-square radii of selected few-body systems. The results are compared to those generated by other methods.

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**Construction of lambda-nucleon s-wave potential through quantum inverse scattering at fixed angular momentum****Author:** Mr. MEOTO, Emile <sup>1</sup>**Co-Author:** Prof. LEKALA, Mantile <sup>2</sup><sup>1</sup> Department of Physics, University of South Africa<sup>2</sup> UNISA**Corresponding Author:** brian.masara@saip.org.za

Quantum systems with a strangeness degree of freedom are very important as they provide an extra dimension, and hence a deeper insight into nuclear matter. Usually phenomenological potentials obtained through meson exchange theories are used in investigating these hypernuclear systems. In this paper potentials for lambda-nucleon interactions in the spin singlet and spin triplet states, constructed through fixed-angular momentum inversion based on Marchenko theory, are presented. Owing to experimental difficulties in obtaining a sufficient number of lambda-nucleon scattering events, theoretical phase shifts are used as input for the inversion. The constructed potential is energy-independent, making it more suitable for quantum-mechanical few-body calculations.